

部分群の計算法

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要約

与えられた有限群について、すべての部分群を求める方法を示す。まず、4次交代群、4次対称群、5次対称群、6次対称群について、すべての部分群を求め、これらを共役類で分類する。これらの部分群のいくつかについては、交換子群を求めることによって、可解であるかどうかを調べる。さらに、与えられた有限群が代数方程式のガロア群である場合に、すべての部分群を求めるところをすべての正規部分群を求めるように変更することにより、ガロア群の組成列を求める。

1. 部分群の計算法

与えられた有限群について、すべての部分群を求める方法を示す。

《計算の準備》

与えられた有限群を $G = \{\sigma_1, \sigma_2, \dots, \sigma_N\}$ (N は位数) とする。各元は数字 $1, 2, \dots, n$ の置換として扱い、これを j_1, j_2, \dots, j_n に移す置換を $\{j_1, j_2, \dots, j_n\}$ と表記する。各元は j_1 の小さい順に、 j_1 が同じならば j_2 の小さい順に、 j_1, j_2 が同じならば j_3 の小さい順に、そして j_1, j_2, \dots, j_{n-2} が同じならば j_{n-1} の小さい順に並べるものとする。

(例) 4次交代群の場合

この場合は $n=4$, $N=12$ で、 G は以下のように表される。

$$G = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6, \sigma_7, \sigma_8, \sigma_9, \sigma_{10}, \sigma_{11}, \sigma_{12}\} \\ = \{\{1, 2, 3, 4\}, \{1, 3, 4, 2\}, \{1, 4, 2, 3\}, \{2, 1, 4, 3\}, \{2, 3, 1, 4\}, \{2, 4, 3, 1\}, \\ \{3, 1, 2, 4\}, \{3, 2, 4, 1\}, \{3, 4, 1, 2\}, \{4, 1, 3, 2\}, \{4, 2, 1, 3\}, \{4, 3, 2, 1\}\}$$

まず、 G の各元について乗積表と逆元を求める。

(例) 4次交代群の場合

以下に $\sigma_4 \sigma_9$ の乗積表を示す。例えば、 $\sigma_4 = \{2, 1, 4, 3\}$ と $\sigma_9 = \{3, 4, 1, 2\}$ の積は $\sigma_{12} = \{4, 3, 2, 1\}$ である。

$i \setminus j$	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	3	1	7	9	8	10	12	11	4	5	6
3	3	1	2	10	11	12	4	6	5	7	9	8
4	4	6	5	1	3	2	11	10	12	8	7	9
5	5	4	6	8	7	9	1	2	3	11	12	10
6	6	5	4	11	12	10	8	9	7	1	3	2
7	7	8	9	2	1	3	5	4	6	12	10	11
8	8	9	7	5	6	4	12	11	10	2	1	3
9	9	7	8	12	10	11	2	3	1	5	6	4
10	10	12	11	3	2	1	9	7	8	6	4	5
11	11	10	12	6	4	5	3	1	2	9	8	7
12	12	11	10	9	8	7	6	5	4	3	2	1

以下に σ_i の逆元 σ_j を示す。例えば、 $\sigma_6 = \{2, 4, 3, 1\}$ の逆元は $\sigma_{10} = \{4, 1, 3, 2\}$ である。

i	1	2	3	4	5	6	7	8	9	10	11	12
j	1	3	2	4	7	10	5	11	9	6	8	12

《部分群の計算》

以下の手順により、すべての部分群を求める。

- (1) 1個の元から生成される部分群を求める。これには、各元から生成される部分群を求め、その中から相異なるものを取り出す。
- (2) 2個の元から生成される部分群を求める。これには、1個の元から生成される部分群のうちの2つを組み合わせて新しい部分群を生成し、その中から既存でないものを取り出す。
- (3) $k=3, 4, 5, \dots$ として、 k 個の元から生成される部分群を求める。これには、 $k-1$ 個の元から生成される部分群と1個の元から生成される部分群を組み合わせて新しい部分群を生成し、その中から既存でないものを取り出す。 k 個の元から生成される部分群がなければ終了とする。

以下に示すのは、Cプログラム風にしたアルゴリズムである。

c : G の部分群を数えるための変数

$H[c]$: G の c 番目の部分群, $H[1:c] : \{H[1], H[2], \dots, H[c]\}$ をまとめたもの

$h[c]$: $H[c]$ の生成元, $h[1:c] : \{h[1], h[2], \dots, h[c]\}$ をまとめたもの

$t[k]$: k 個の元から生成される部分群の最後尾の番号

手順(1)のアルゴリズムを示す。1個の元 $\{\sigma_i\}$ から生成される部分群 $\langle \sigma_i \rangle$ を求める。これが新規の部分群ならば、これを $h[c], H[c]$ に代入する。

```

c=0;
for(i=1; i<=N; i++) {
    if( $\langle \sigma_i \rangle \in H[1:c]$ ) continue;
    c++; h[c]= $\{\sigma_i\}$ ; H[c]= $\langle \sigma_i \rangle$ ;
}
t[1]=c;

```

手順(2)のアルゴリズムを示す。1個の元 $h[i]$ から生成される部分群 $H[i]$ と、同じく1個の元 $h[j]$ から生成される部分群 $H[j]$ を合成することによって、2個の元 $h[i] \cup h[j]$ から生成される部分群 $\langle H[i], H[j] \rangle$ を求める。これが新規の部分群ならば、これを $h[c], H[c]$ に代入する。2個の元から生成される部分群が1つもなければ($t[2]=t[1]$)、これで計算を終了する。

```

for(i=2; i<=t[1]-1; i++) {
    for(j=i+1; j<=t[1]; j++) {
        if( $\langle H[i], H[j] \rangle \in H[1:c]$ ) continue;
        c++; h[c]= $h[i] \cup h[j]$ ; H[c]= $\langle H[i], H[j] \rangle$ ;
    }
}
t[2]=c;

```

手順(3)のアルゴリズムを示す。 $k=3, 4, 5, \dots$ として、 $k-1$ 個の元 $h[i]$ から生成される部分群 $H[i]$ と、1個の元 $h[j]$ から生成される部分群 $H[j]$ を合成することによって、 k 個の元 $h[i] \cup h[j]$ から生成される部分群 $\langle H[i], H[j] \rangle$ を求める。これが新規の部分群ならば、これを $h[c], H[c]$ に代入する。 k 個の元から生成される部分群が1つもなければ($t[k]=t[k-1]$)、これで計算を終了する。

```

for (k=3; t[k-1]>t[k-2]; k++) {
  for (i=t[k-2]+1; i<=t[k-1]; i++) {
    for (j=2; j<=t[1]; j++) {
      if (max(h[i])>h[j]) continue;
      if (<H[i], H[j]>∈H[1:c]) continue;
      c++; h[c]=h[i] ∪ h[j]; H[c]=<H[i], H[j]>;
    }
  }
  t[k]=c;
}

```

(例) 4次交代群の場合

1個の元から生成される部分群は以下の8個である。

- $H_1 = \langle \sigma_1 \rangle = \{ \sigma_1 \}$: 単位群, 位数1
- $H_2 = \langle \sigma_2 \rangle = \{ \sigma_1, \sigma_2, \sigma_3 \}$: (2, 3, 4)の偶置換, 位数3
- $H_3 = \langle \sigma_4 \rangle = \{ \sigma_1, \sigma_4 \}$: (1, 2)の置換と(3, 4)の置換を同時に行ったもの, 位数2
- $H_4 = \langle \sigma_5 \rangle = \{ \sigma_1, \sigma_5, \sigma_7 \}$: (1, 2, 3)の偶置換, 位数3
- $H_5 = \langle \sigma_6 \rangle = \{ \sigma_1, \sigma_6, \sigma_{10} \}$: (1, 2, 4)の偶置換, 位数3
- $H_6 = \langle \sigma_8 \rangle = \{ \sigma_1, \sigma_8, \sigma_{11} \}$: (1, 3, 4)の偶置換, 位数3
- $H_7 = \langle \sigma_9 \rangle = \{ \sigma_1, \sigma_9 \}$: (1, 3)の置換と(2, 4)の置換を同時に行ったもの, 位数2
- $H_8 = \langle \sigma_{12} \rangle = \{ \sigma_1, \sigma_{12} \}$: (1, 4)の置換と(2, 3)の置換を同時に行ったもの, 位数2

2個の元から生成される部分群は以下の2個である。

- $H_9 = \langle \sigma_2, \sigma_4 \rangle = \{ \sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6, \sigma_7, \sigma_8, \sigma_9, \sigma_{10}, \sigma_{11}, \sigma_{12} \}$: (1, 2, 3, 4)の偶置換, 位数12
- $H_{10} = \langle \sigma_4, \sigma_9 \rangle = \{ \sigma_1, \sigma_4, \sigma_9, \sigma_{12} \}$: $\{ \{1, 2, 3, 4\}, \{2, 1, 4, 3\}, \{3, 4, 1, 2\}, \{4, 3, 2, 1\} \}$, 位数4

3個の元から生成される部分群はない。以上より4次交代群の部分群は10個である。

《共役類の計算》

G の部分群 H に対して, $gHg^{-1} (g \in G)$ で表される部分群を H の共役類という。 G の部分群は共役類で分類することができる。以下に示すのは, k 番目の部分群 $H[k]$ の共役類を求め, その番号を配列 S に代入するアルゴリズムである。sortは番号の小さい順に並べ替える関数である。

```

S = {};
for (i=1; i<=N; i++) {
  T = σiH[k]σi-1;
  T = sort(T);
  for (j=1; j<=c; j++) {
    if (H[j] == T) break;
  }
  if (j ∉ S) S = S ∪ j;
}
S = sort(S);

```

(例) 4次交代群の場合

以下の5種の共役類に分類できる。

$$\{H_1\}, \{H_2, H_4, H_5, H_6\}, \{H_3, H_7, H_8\}, \{H_9\}, \{H_{10}\}$$

4次交代群の部分群を共役類で分類して下表に示す。

位数	部分群
1	$H_1 = \langle \sigma_1 \rangle = \{\sigma_1\}$
2	$H_3 = \langle \sigma_4 \rangle = \{\sigma_1, \sigma_4\}, H_7 = \langle \sigma_9 \rangle = \{\sigma_1, \sigma_9\}, H_8 = \langle \sigma_{12} \rangle = \{\sigma_1, \sigma_{12}\}$
3	$H_2 = \langle \sigma_2 \rangle = \{\sigma_1, \sigma_2, \sigma_3\}, H_4 = \langle \sigma_5 \rangle = \{\sigma_1, \sigma_5, \sigma_7\}, H_5 = \langle \sigma_6 \rangle = \{\sigma_1, \sigma_6, \sigma_{10}\}, H_6 = \langle \sigma_8 \rangle = \{\sigma_1, \sigma_8, \sigma_{11}\}$
4	$H_{10} = \langle \sigma_4, \sigma_9 \rangle = \{\sigma_1, \sigma_4, \sigma_9, \sigma_{12}\}$
12	$H_9 = \langle \sigma_2, \sigma_4 \rangle = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6, \sigma_7, \sigma_8, \sigma_9, \sigma_{10}, \sigma_{11}, \sigma_{12}\}$

2. 4次対称群の部分群

この場合は $n=4$, $N=24$ で, G は以下のように表される。

$$\begin{aligned} G = & \{\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6, \sigma_7, \sigma_8, \sigma_9, \sigma_{10}, \sigma_{11}, \sigma_{12}, \sigma_{13}, \sigma_{14}, \sigma_{15}, \sigma_{16}, \sigma_{17}, \sigma_{18}, \sigma_{19}, \sigma_{20}, \sigma_{21}, \sigma_{22}, \sigma_{23}, \sigma_{24}\} \\ = & \{\{1, 2, 3, 4\}, \{1, 2, 4, 3\}, \{1, 3, 2, 4\}, \{1, 3, 4, 2\}, \{1, 4, 2, 3\}, \{1, 4, 3, 2\}, \\ & \{2, 1, 3, 4\}, \{2, 1, 4, 3\}, \{2, 3, 1, 4\}, \{2, 3, 4, 1\}, \{2, 4, 1, 3\}, \{2, 4, 3, 1\}, \\ & \{3, 1, 2, 4\}, \{3, 1, 4, 2\}, \{3, 2, 1, 4\}, \{3, 2, 4, 1\}, \{3, 4, 1, 2\}, \{3, 4, 2, 1\}, \\ & \{4, 1, 2, 3\}, \{4, 1, 3, 2\}, \{4, 2, 1, 3\}, \{4, 2, 3, 1\}, \{4, 3, 1, 2\}, \{4, 3, 2, 1\}\} \end{aligned}$$

1個の元から生成される部分群は以下の17個である。

$$\begin{aligned} H_1 = \langle \sigma_1 \rangle &= \{\sigma_1\} : \text{単位群, 位数1} \\ H_2 = \langle \sigma_2 \rangle &= \{\sigma_1, \sigma_2\} : (3, 4) \text{の置換, 位数2} \\ H_3 = \langle \sigma_3 \rangle &= \{\sigma_1, \sigma_3\} : (2, 3) \text{の置換, 位数2} \\ H_4 = \langle \sigma_4 \rangle &= \{\sigma_1, \sigma_4, \sigma_5\} : (2, 3, 4) \text{の偶置換, 位数3} \\ H_5 = \langle \sigma_6 \rangle &= \{\sigma_1, \sigma_6\} : (2, 4) \text{の置換, 位数2} \\ H_6 = \langle \sigma_7 \rangle &= \{\sigma_1, \sigma_7\} : (1, 2) \text{の置換, 位数2} \\ H_7 = \langle \sigma_8 \rangle &= \{\sigma_1, \sigma_8\} : (1, 2) \text{の置換と}(3, 4) \text{の置換を同時に行ったもの, 位数2} \\ H_8 = \langle \sigma_9 \rangle &= \{\sigma_1, \sigma_9, \sigma_{13}\} : (1, 2, 3) \text{の偶置換, 位数3} \\ H_9 = \langle \sigma_{10} \rangle &= \{\sigma_1, \sigma_{10}, \sigma_{17}, \sigma_{19}\} : (1, 2, 3, 4) \text{の巡回置換, 位数4} \\ H_{10} = \langle \sigma_{11} \rangle &= \{\sigma_1, \sigma_{11}, \sigma_{14}, \sigma_{24}\} : (1, 2, 4, 3) \text{の巡回置換, 位数4} \\ H_{11} = \langle \sigma_{12} \rangle &= \{\sigma_1, \sigma_{12}, \sigma_{20}\} : (1, 2, 4) \text{の偶置換, 位数3} \\ H_{12} = \langle \sigma_{15} \rangle &= \{\sigma_1, \sigma_{15}\} : (1, 3) \text{の置換, 位数2} \\ H_{13} = \langle \sigma_{16} \rangle &= \{\sigma_1, \sigma_{16}, \sigma_{21}\} : (1, 3, 4) \text{の偶置換, 位数3} \\ H_{14} = \langle \sigma_{17} \rangle &= \{\sigma_1, \sigma_{17}\} : (1, 3) \text{の置換と}(2, 4) \text{の置換を同時に行ったもの, 位数2} \\ H_{15} = \langle \sigma_{18} \rangle &= \{\sigma_1, \sigma_8, \sigma_{18}, \sigma_{23}\} : (1, 3, 2, 4) \text{の巡回置換, 位数4} \\ H_{16} = \langle \sigma_{22} \rangle &= \{\sigma_1, \sigma_{22}\} : (1, 4) \text{の置換, 位数2} \\ H_{17} = \langle \sigma_{24} \rangle &= \{\sigma_1, \sigma_{24}\} : (1, 4) \text{の置換と}(2, 3) \text{の置換を同時に行ったもの, 位数2} \end{aligned}$$

2個の元から生成される部分群は以下の13個である。

$$\begin{aligned} H_{18} = \langle \sigma_2, \sigma_3 \rangle &= \{\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6\} : (2, 3, 4) \text{の全置換, 位数6} \\ H_{19} = \langle \sigma_2, \sigma_7 \rangle &= \{\sigma_1, \sigma_2, \sigma_7, \sigma_8\} : (1, 2) \text{の置換と}(3, 4) \text{の置換の積, 位数4} \\ H_{20} = \langle \sigma_2, \sigma_9 \rangle &= \{\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6, \sigma_7, \sigma_8, \sigma_9, \sigma_{10}, \sigma_{11}, \sigma_{12}, \sigma_{13}, \sigma_{14}, \sigma_{15}, \sigma_{16}, \sigma_{17}, \sigma_{18}, \sigma_{19}, \sigma_{20}, \sigma_{21}, \dots, \sigma_{24}\} : \\ & (1, 2, 3, 4) \text{の全置換, 位数24} \\ H_{21} = \langle \sigma_2, \sigma_{15} \rangle &= \{\sigma_1, \sigma_2, \sigma_{15}, \sigma_{16}, \sigma_{21}, \sigma_{22}\} : (1, 3, 4) \text{の全置換, 位数6} \\ H_{22} = \langle \sigma_2, \sigma_{17} \rangle &= \{\sigma_1, \sigma_2, \sigma_7, \sigma_8, \sigma_{17}, \sigma_{18}, \sigma_{23}, \sigma_{24}\} : (1, 3, 2, 4) \text{の巡回置換と}(3, 4) \text{の置換の積, 位数8} \\ H_{23} = \langle \sigma_3, \sigma_7 \rangle &= \{\sigma_1, \sigma_3, \sigma_7, \sigma_9, \sigma_{13}, \sigma_{15}\} : (1, 2, 3) \text{の全置換, 位数6} \\ H_{24} = \langle \sigma_3, \sigma_8 \rangle &= \{\sigma_1, \sigma_3, \sigma_8, \sigma_{11}, \sigma_{14}, \sigma_{17}, \sigma_{22}, \sigma_{24}\} : (1, 2, 4, 3) \text{の巡回置換と}(2, 3) \text{の置換の積, 位数8} \\ H_{25} = \langle \sigma_3, \sigma_{22} \rangle &= \{\sigma_1, \sigma_3, \sigma_{22}, \sigma_{24}\} : (1, 4) \text{の置換と}(2, 3) \text{の置換の積, 位数4} \\ H_{26} = \langle \sigma_4, \sigma_8 \rangle &= \{\sigma_1, \sigma_4, \sigma_5, \sigma_8, \sigma_9, \sigma_{12}, \sigma_{13}, \sigma_{16}, \sigma_{17}, \sigma_{20}, \sigma_{21}, \sigma_{24}\} : (1, 2, 3, 4) \text{の偶置換, 位数12} \\ H_{27} = \langle \sigma_6, \sigma_7 \rangle &= \{\sigma_1, \sigma_6, \sigma_7, \sigma_{12}, \sigma_{20}, \sigma_{22}\} : (1, 2, 4) \text{の全置換, 位数6} \\ H_{28} = \langle \sigma_6, \sigma_8 \rangle &= \{\sigma_1, \sigma_6, \sigma_8, \sigma_{10}, \sigma_{15}, \sigma_{17}, \sigma_{19}, \sigma_{24}\} : (1, 2, 3, 4) \text{の巡回置換と}(2, 4) \text{の置換の積, 位数8} \\ H_{29} = \langle \sigma_6, \sigma_{15} \rangle &= \{\sigma_1, \sigma_6, \sigma_{15}, \sigma_{17}\} : (1, 3) \text{の置換と}(2, 4) \text{の置換の積, 位数4} \\ H_{30} = \langle \sigma_8, \sigma_{17} \rangle &= \{\sigma_1, \sigma_8, \sigma_{17}, \sigma_{24}\} : \{\{1, 2, 3, 4\}, \{2, 1, 4, 3\}, \{3, 4, 1, 2\}, \{4, 3, 2, 1\}\}, \text{位数4} \end{aligned}$$

3個の元から生成される部分群はない。以上より4次対称群の部分群は30個である。4次対称群の部分群を共役類で分類して下表に示す。これは参考文献(1)の2頁の表と一致する。

位数	部分群
1	$H_1 = \langle \sigma_1 \rangle = \{ \sigma_1 \}$
2	$H_2 = \langle \sigma_2 \rangle = \{ \sigma_1, \sigma_2 \}, H_3 = \langle \sigma_3 \rangle = \{ \sigma_1, \sigma_3 \}, H_5 = \langle \sigma_6 \rangle = \{ \sigma_1, \sigma_6 \}, H_6 = \langle \sigma_7 \rangle = \{ \sigma_1, \sigma_7 \},$ $H_{12} = \langle \sigma_{15} \rangle = \{ \sigma_1, \sigma_{15} \}, H_{16} = \langle \sigma_{22} \rangle = \{ \sigma_1, \sigma_{22} \}$ $H_7 = \langle \sigma_8 \rangle = \{ \sigma_1, \sigma_8 \}, H_{14} = \langle \sigma_{17} \rangle = \{ \sigma_1, \sigma_{17} \}, H_{17} = \langle \sigma_{24} \rangle = \{ \sigma_1, \sigma_{24} \}$
3	$H_4 = \langle \sigma_4 \rangle = \{ \sigma_1, \sigma_4, \sigma_5 \}, H_8 = \langle \sigma_9 \rangle = \{ \sigma_1, \sigma_9, \sigma_{13} \}, H_{11} = \langle \sigma_{12} \rangle = \{ \sigma_1, \sigma_{12}, \sigma_{20} \},$ $H_{13} = \langle \sigma_{16} \rangle = \{ \sigma_1, \sigma_{16}, \sigma_{21} \}$
4	$H_9 = \langle \sigma_{10} \rangle = \{ \sigma_1, \sigma_{10}, \sigma_{17}, \sigma_{19} \}, H_{10} = \langle \sigma_{11} \rangle = \{ \sigma_1, \sigma_{11}, \sigma_{14}, \sigma_{24} \}, H_{15} = \langle \sigma_{18} \rangle = \{ \sigma_1, \sigma_8, \sigma_{18}, \sigma_{23} \}$ $H_{19} = \langle \sigma_2, \sigma_7 \rangle = \{ \sigma_1, \sigma_2, \sigma_7, \sigma_8 \}, H_{25} = \langle \sigma_3, \sigma_{22} \rangle = \{ \sigma_1, \sigma_3, \sigma_{22}, \sigma_{24} \},$ $H_{29} = \langle \sigma_6, \sigma_{15} \rangle = \{ \sigma_1, \sigma_6, \sigma_{15}, \sigma_{17} \}$ $H_{30} = \langle \sigma_8, \sigma_{17} \rangle = \{ \sigma_1, \sigma_8, \sigma_{17}, \sigma_{24} \}$
6	$H_{18} = \langle \sigma_2, \sigma_3 \rangle = \{ \sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6 \}, H_{21} = \langle \sigma_2, \sigma_{15} \rangle = \{ \sigma_1, \sigma_2, \sigma_{15}, \sigma_{16}, \sigma_{21}, \sigma_{22} \},$ $H_{23} = \langle \sigma_3, \sigma_7 \rangle = \{ \sigma_1, \sigma_3, \sigma_7, \sigma_9, \sigma_{13}, \sigma_{15} \}, H_{27} = \langle \sigma_6, \sigma_7 \rangle = \{ \sigma_1, \sigma_6, \sigma_7, \sigma_{12}, \sigma_{20}, \sigma_{22} \}$
8	$H_{22} = \langle \sigma_2, \sigma_{17} \rangle = \{ \sigma_1, \sigma_2, \sigma_7, \sigma_8, \sigma_{17}, \sigma_{18}, \sigma_{23}, \sigma_{24} \},$ $H_{24} = \langle \sigma_3, \sigma_8 \rangle = \{ \sigma_1, \sigma_3, \sigma_8, \sigma_{11}, \sigma_{14}, \sigma_{17}, \sigma_{22}, \sigma_{24} \},$ $H_{28} = \langle \sigma_6, \sigma_8 \rangle = \{ \sigma_1, \sigma_6, \sigma_8, \sigma_{10}, \sigma_{15}, \sigma_{17}, \sigma_{19}, \sigma_{24} \}$
12	$H_{26} = \langle \sigma_4, \sigma_8 \rangle = \{ \sigma_1, \sigma_4, \sigma_5, \sigma_8, \sigma_9, \sigma_{12}, \sigma_{13}, \sigma_{16}, \sigma_{17}, \sigma_{20}, \sigma_{21}, \sigma_{24} \}$
24	$H_{20} = \langle \sigma_2, \sigma_9 \rangle = \{ \sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6, \sigma_7, \sigma_8, \sigma_9, \sigma_{10}, \sigma_{11}, \sigma_{12}, \sigma_{13}, \sigma_{14}, \sigma_{15}, \sigma_{16}, \sigma_{17}, \sigma_{18}, \dots, \sigma_{24} \}$

3. 5次対称群の部分群

この場合は $n=5$, $N=120$ で, G は以下のように表される。(途中は省略)

$$\begin{aligned} G &= \{\sigma_1, \dots, \sigma_{25}, \dots, \sigma_{49}, \dots, \sigma_{73}, \dots, \sigma_{97}, \dots, \sigma_{120}\} \\ &= \{\{1, 2, 3, 4, 5\}, \dots, \{2, 1, 3, 4, 5\}, \dots, \{3, 1, 2, 4, 5\}, \dots, \{4, 1, 2, 3, 5\}, \dots, \\ &\quad \{5, 1, 2, 3, 4\}, \dots, \{5, 4, 3, 2, 1\}\} \end{aligned}$$

1個の元から生成される部分群は以下の67個である。ただし, 同一の共役類に属する部分群は, 代表の1個(最初に見つかったもの)だけを示す。

$$\begin{aligned} H_1 &= \langle \sigma_1 \rangle = \{\sigma_1\} : \text{単位群, 位数1} \\ H_2 &= \langle \sigma_2 \rangle = \{\sigma_1, \sigma_2\} : (4, 5) \text{の置換, 位数2, 共役類は計10個} \\ H_4 &= \langle \sigma_4 \rangle = \{\sigma_1, \sigma_4, \sigma_5\} : (3, 4, 5) \text{の偶置換, 位数3, 共役類は計10個} \\ H_7 &= \langle \sigma_8 \rangle = \{\sigma_1, \sigma_8\} : (2, 3) \text{の置換と}(4, 5) \text{の置換を同時に行ったもの, 位数2, 共役類は計15個} \\ H_9 &= \langle \sigma_{10} \rangle = \{\sigma_1, \sigma_{10}, \sigma_{17}, \sigma_{19}\} : (2, 3, 4, 5) \text{の巡回置換, 位数4, 共役類は計15個} \\ H_{21} &= \langle \sigma_{28} \rangle = \{\sigma_1, \sigma_4, \sigma_5, \sigma_{25}, \sigma_{28}, \sigma_{29}\} : (1, 2) \text{の置換と}(3, 4, 5) \text{の偶置換の積, 位数6, 共役類は計10個} \\ H_{26} &= \langle \sigma_{34} \rangle = \{\sigma_1, \sigma_{34}, \sigma_{65}, \sigma_{91}, \sigma_{97}\} : (1, 2, 3, 4, 5) \text{の巡回置換, 位数5, 共役類は計6個} \end{aligned}$$

2個の元から生成される部分群は以下の89個である。

$$\begin{aligned} H_{68} &= \langle \sigma_2, \sigma_3 \rangle = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6\} : (3, 4, 5) \text{の全置換, 位数6, 共役類は計10個} \\ H_{69} &= \langle \sigma_2, \sigma_7 \rangle = \{\sigma_1, \sigma_2, \sigma_7, \sigma_8\} : (2, 3) \text{の置換と}(4, 5) \text{の置換の積, 位数4, 共役類は計15個} \\ H_{70} &= \langle \sigma_2, \sigma_9 \rangle = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6, \sigma_7, \sigma_8, \sigma_9, \sigma_{10}, \sigma_{11}, \sigma_{12}, \sigma_{13}, \sigma_{14}, \sigma_{15}, \sigma_{16}, \sigma_{17}, \sigma_{18}, \sigma_{19}, \sigma_{20}, \sigma_{21}, \dots, \sigma_{24}\} : \\ &\quad (2, 3, 4, 5) \text{の全置換, 位数24, 共役類は計5個} \\ H_{72} &= \langle \sigma_2, \sigma_{17} \rangle = \{\sigma_1, \sigma_2, \sigma_7, \sigma_8, \sigma_{17}, \sigma_{18}, \sigma_{23}, \sigma_{24}\} : (2, 4, 3, 5) \text{の巡回置換と}(4, 5) \text{の置換の積, 位数8, 共役類は計15個} \\ H_{74} &= \langle \sigma_2, \sigma_{27} \rangle = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6, \sigma_{25}, \sigma_{26}, \sigma_{27}, \sigma_{28}, \sigma_{29}, \sigma_{30}\} : (1, 2) \text{の置換と}(3, 4, 5) \text{の全置換の積, 位数12, 共役類は計10個} \\ H_{75} &= \langle \sigma_2, \sigma_{33} \rangle = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6, \sigma_7, \sigma_8, \sigma_9, \sigma_{10}, \sigma_{11}, \sigma_{12}, \sigma_{13}, \sigma_{14}, \sigma_{15}, \sigma_{16}, \sigma_{17}, \sigma_{18}, \sigma_{19}, \sigma_{20}, \dots, \sigma_{120}\} : \\ &\quad (1, 2, 3, 4, 5) \text{の全置換, 位数120} \\ H_{95} &= \langle \sigma_4, \sigma_8 \rangle = \{\sigma_1, \sigma_4, \sigma_5, \sigma_8, \sigma_9, \sigma_{12}, \sigma_{13}, \sigma_{16}, \sigma_{17}, \sigma_{20}, \sigma_{21}, \sigma_{24}\} : (2, 3, 4, 5) \text{の偶置換, 位数12, 共役類は計5個} \\ H_{96} &= \langle \sigma_4, \sigma_{26} \rangle = \{\sigma_1, \sigma_4, \sigma_5, \sigma_{26}, \sigma_{27}, \sigma_{30}\} : (3, 4, 5) \text{の全置換で, そのうちの奇置換では}(1, 2) \text{を交換したものの, 位数6, 共役類は計10個} \\ H_{97} &= \langle \sigma_4, \sigma_{31} \rangle = \{\sigma_1, \sigma_4, \sigma_5, \sigma_8, \sigma_9, \sigma_{12}, \sigma_{13}, \sigma_{16}, \sigma_{17}, \sigma_{20}, \sigma_{21}, \sigma_{24}, \sigma_{26}, \sigma_{27}, \sigma_{30}, \sigma_{31}, \sigma_{34}, \sigma_{35}, \sigma_{38}, \sigma_{39}, \dots, \sigma_{120}\} : \\ &\quad (1, 2, 3, 4, 5) \text{の偶置換, 位数60} \\ H_{116} &= \langle \sigma_8, \sigma_{17} \rangle = \{\sigma_1, \sigma_8, \sigma_{17}, \sigma_{24}\} : \{\{1, 2, 3, 4, 5\}, \{1, 3, 2, 5, 4\}, \{1, 4, 5, 2, 3\}, \{1, 5, 4, 3, 2\}\}, \text{位数4, 共役類は計5個} \\ H_{118} &= \langle \sigma_8, \sigma_{27} \rangle = \{\sigma_1, \sigma_8, \sigma_{27}, \sigma_{38}, \sigma_{53}, \sigma_{68}, \sigma_{83}, \sigma_{94}, \sigma_{113}, \sigma_{120}\} : (2, 3) \text{の置換と}(4, 5) \text{の置換を同時に行ったものと}(1, 2, 4, 5, 3) \text{の巡回置換の積, 位数10, 共役類は計6個} \\ H_{120} &= \langle \sigma_8, \sigma_{33} \rangle = \{\sigma_1, \sigma_8, \sigma_{18}, \sigma_{23}, \sigma_{30}, \sigma_{33}, \sigma_{40}, \sigma_{43}, \sigma_{52}, \sigma_{59}, \sigma_{61}, \sigma_{70}, \sigma_{73}, \sigma_{80}, \sigma_{90}, \sigma_{95}, \sigma_{99}, \sigma_{108}, \sigma_{110}, \sigma_{117}\} : \\ &\quad (1, 2, 5, 4, 3) \text{の巡回置換と}(2, 4, 3, 5) \text{の巡回置換の積, 位数20, 共役類は計6個} \end{aligned}$$

3個の元から生成される部分群はない。以上より5次対称群の部分群は156個である。5次対称群の部分群を共役類で分類して下表に示す。これは参考文献(1)の3頁の表と一致する。

位数	部分群
1	$H_1 = \langle \sigma_1 \rangle$
2	$H_2 = \langle \sigma_2 \rangle, H_3 = \langle \sigma_3 \rangle, H_5 = \langle \sigma_6 \rangle, H_6 = \langle \sigma_7 \rangle, H_{12} = \langle \sigma_{15} \rangle, H_{16} = \langle \sigma_{22} \rangle, H_{18} = \langle \sigma_{25} \rangle, H_{41} = \langle \sigma_{55} \rangle, H_{54} = \langle \sigma_{81} \rangle, H_{63} = \langle \sigma_{106} \rangle$ $H_7 = \langle \sigma_8 \rangle, H_{14} = \langle \sigma_{17} \rangle, H_{17} = \langle \sigma_{24} \rangle, H_{19} = \langle \sigma_{26} \rangle, H_{20} = \langle \sigma_{27} \rangle, H_{22} = \langle \sigma_{30} \rangle, H_{42} = \langle \sigma_{56} \rangle, H_{47} = \langle \sigma_{61} \rangle, H_{51} = \langle \sigma_{68} \rangle,$ $H_{56} = \langle \sigma_{83} \rangle, H_{58} = \langle \sigma_{87} \rangle, H_{61} = \langle \sigma_{95} \rangle, H_{64} = \langle \sigma_{108} \rangle, H_{65} = \langle \sigma_{112} \rangle, H_{67} = \langle \sigma_{120} \rangle$
3	$H_4 = \langle \sigma_4 \rangle, H_8 = \langle \sigma_9 \rangle, H_{11} = \langle \sigma_{12} \rangle, H_{13} = \langle \sigma_{16} \rangle, H_{23} = \langle \sigma_{31} \rangle, H_{31} = \langle \sigma_{39} \rangle, H_{38} = \langle \sigma_{46} \rangle, H_{43} = \langle \sigma_{57} \rangle, H_{46} = \langle \sigma_{60} \rangle,$ $H_{55} = \langle \sigma_{82} \rangle$
4	$H_9 = \langle \sigma_{10} \rangle, H_{10} = \langle \sigma_{11} \rangle, H_{15} = \langle \sigma_{18} \rangle, H_{25} = \langle \sigma_{33} \rangle, H_{28} = \langle \sigma_{36} \rangle, H_{29} = \langle \sigma_{37} \rangle, H_{32} = \langle \sigma_{40} \rangle, H_{36} = \langle \sigma_{44} \rangle, H_{37} = \langle \sigma_{45} \rangle,$ $H_{44} = \langle \sigma_{58} \rangle, H_{45} = \langle \sigma_{59} \rangle, H_{49} = \langle \sigma_{63} \rangle, H_{52} = \langle \sigma_{70} \rangle, H_{57} = \langle \sigma_{84} \rangle, H_{62} = \langle \sigma_{96} \rangle$ $H_{69} = \langle \sigma_2, \sigma_7 \rangle, H_{73} = \langle \sigma_2, \sigma_{25} \rangle, H_{77} = \langle \sigma_2, \sigma_{55} \rangle, H_{86} = \langle \sigma_3, \sigma_{22} \rangle, H_{87} = \langle \sigma_3, \sigma_{25} \rangle, H_{93} = \langle \sigma_3, \sigma_{106} \rangle, H_{101} = \langle \sigma_6, \sigma_{15} \rangle,$ $H_{102} = \langle \sigma_6, \sigma_{25} \rangle, H_{108} = \langle \sigma_6, \sigma_{81} \rangle, H_{114} = \langle \sigma_7, \sigma_{81} \rangle, H_{115} = \langle \sigma_7, \sigma_{106} \rangle, H_{135} = \langle \sigma_{15}, \sigma_{55} \rangle, H_{136} = \langle \sigma_{15}, \sigma_{106} \rangle,$ $H_{147} = \langle \sigma_{22}, \sigma_{55} \rangle, H_{148} = \langle \sigma_{22}, \sigma_{81} \rangle$ $H_{116} = \langle \sigma_8, \sigma_{17} \rangle, H_{153} = \langle \sigma_{26}, \sigma_{95} \rangle, H_{154} = \langle \sigma_{27}, \sigma_{61} \rangle, H_{155} = \langle \sigma_{30}, \sigma_{68} \rangle, H_{156} = \langle \sigma_{56}, \sigma_{83} \rangle$
5	$H_{26} = \langle \sigma_{34} \rangle, H_{27} = \langle \sigma_{35} \rangle, H_{30} = \langle \sigma_{38} \rangle, H_{34} = \langle \sigma_{42} \rangle, H_{35} = \langle \sigma_{43} \rangle, H_{39} = \langle \sigma_{47} \rangle$
6	$H_{21} = \langle \sigma_{28} \rangle, H_{24} = \langle \sigma_{32} \rangle, H_{33} = \langle \sigma_{41} \rangle, H_{40} = \langle \sigma_{48} \rangle, H_{48} = \langle \sigma_{62} \rangle, H_{50} = \langle \sigma_{66} \rangle, H_{53} = \langle \sigma_{71} \rangle, H_{59} = \langle \sigma_{88} \rangle, H_{60} = \langle \sigma_{89} \rangle,$ $H_{66} = \langle \sigma_{114} \rangle$ $H_{68} = \langle \sigma_2, \sigma_3 \rangle, H_{71} = \langle \sigma_2, \sigma_{15} \rangle, H_{80} = \langle \sigma_2, \sigma_{81} \rangle, H_{84} = \langle \sigma_3, \sigma_7 \rangle, H_{89} = \langle \sigma_3, \sigma_{55} \rangle, H_{99} = \langle \sigma_6, \sigma_7 \rangle, H_{104} = \langle \sigma_6, \sigma_{55} \rangle,$ $H_{110} = \langle \sigma_7, \sigma_{25} \rangle, H_{131} = \langle \sigma_{15}, \sigma_{25} \rangle, H_{143} = \langle \sigma_{22}, \sigma_{25} \rangle$ $H_{96} = \langle \sigma_4, \sigma_{26} \rangle, H_{117} = \langle \sigma_8, \sigma_{26} \rangle, H_{122} = \langle \sigma_8, \sigma_{82} \rangle, H_{124} = \langle \sigma_9, \sigma_{108} \rangle, H_{130} = \langle \sigma_{12}, \sigma_{83} \rangle, H_{138} = \langle \sigma_{16}, \sigma_{56} \rangle,$ $H_{141} = \langle \sigma_{17}, \sigma_{30} \rangle, H_{142} = \langle \sigma_{17}, \sigma_{60} \rangle, H_{150} = \langle \sigma_{24}, \sigma_{27} \rangle, H_{152} = \langle \sigma_{24}, \sigma_{57} \rangle$
8	$H_{72} = \langle \sigma_2, \sigma_{17} \rangle, H_{81} = \langle \sigma_2, \sigma_{83} \rangle, H_{83} = \langle \sigma_2, \sigma_{95} \rangle, H_{85} = \langle \sigma_3, \sigma_8 \rangle, H_{90} = \langle \sigma_3, \sigma_{56} \rangle, H_{91} = \langle \sigma_3, \sigma_{61} \rangle, H_{100} = \langle \sigma_6, \sigma_8 \rangle,$ $H_{105} = \langle \sigma_6, \sigma_{56} \rangle, H_{107} = \langle \sigma_6, \sigma_{68} \rangle, H_{112} = \langle \sigma_7, \sigma_{27} \rangle, H_{113} = \langle \sigma_7, \sigma_{30} \rangle, H_{132} = \langle \sigma_{15}, \sigma_{26} \rangle, H_{133} = \langle \sigma_{15}, \sigma_{27} \rangle,$ $H_{144} = \langle \sigma_{22}, \sigma_{26} \rangle, H_{146} = \langle \sigma_{22}, \sigma_{30} \rangle$
10	$H_{118} = \langle \sigma_8, \sigma_{27} \rangle, H_{119} = \langle \sigma_8, \sigma_{30} \rangle, H_{139} = \langle \sigma_{17}, \sigma_{26} \rangle, H_{140} = \langle \sigma_{17}, \sigma_{27} \rangle, H_{149} = \langle \sigma_{24}, \sigma_{26} \rangle, H_{151} = \langle \sigma_{24}, \sigma_{30} \rangle$
12	$H_{74} = \langle \sigma_2, \sigma_{27} \rangle, H_{79} = \langle \sigma_2, \sigma_{61} \rangle, H_{82} = \langle \sigma_2, \sigma_{87} \rangle, H_{92} = \langle \sigma_3, \sigma_{68} \rangle, H_{94} = \langle \sigma_3, \sigma_{112} \rangle, H_{106} = \langle \sigma_6, \sigma_{61} \rangle, H_{109} = \langle \sigma_6, \sigma_{87} \rangle,$ $H_{111} = \langle \sigma_7, \sigma_{26} \rangle, H_{134} = \langle \sigma_{15}, \sigma_{30} \rangle, H_{145} = \langle \sigma_{22}, \sigma_{27} \rangle$ $H_{95} = \langle \sigma_4, \sigma_8 \rangle, H_{98} = \langle \sigma_4, \sigma_{56} \rangle, H_{123} = \langle \sigma_9, \sigma_{27} \rangle, H_{129} = \langle \sigma_{12}, \sigma_{30} \rangle, H_{137} = \langle \sigma_{16}, \sigma_{26} \rangle$
20	$H_{120} = \langle \sigma_8, \sigma_{33} \rangle, H_{121} = \langle \sigma_8, \sigma_{36} \rangle, H_{125} = \langle \sigma_{10}, \sigma_{26} \rangle, H_{126} = \langle \sigma_{10}, \sigma_{27} \rangle, H_{127} = \langle \sigma_{11}, \sigma_{26} \rangle, H_{128} = \langle \sigma_{11}, \sigma_{30} \rangle$
24	$H_{70} = \langle \sigma_2, \sigma_9 \rangle, H_{76} = \langle \sigma_2, \sigma_{39} \rangle, H_{78} = \langle \sigma_2, \sigma_{57} \rangle, H_{88} = \langle \sigma_3, \sigma_{31} \rangle, H_{103} = \langle \sigma_6, \sigma_{31} \rangle$
60	$H_{97} = \langle \sigma_4, \sigma_{31} \rangle$
120	$H_{75} = \langle \sigma_2, \sigma_{33} \rangle$

《交換子群の計算》

$D(G) = \langle \{\sigma^{-1}\tau^{-1}\sigma\tau \mid \sigma, \tau \in G\} \rangle$ で表される群を G の交換子群という。 G が可解であるための条件は、交換子群の列 $D(G), D^2(G), D^3(G), \dots$ を求めたとき、列の最後が単位群になることである。以下に示すのは、 k 番目の部分群 $H[k]$ の交換子群の列を求め、その番号を配列 S に代入するアルゴリズムである。

```

S={k};
l=k;
while(true) {
  T=<{\sigma^{-1}\tau^{-1}\sigma\tau \mid \sigma, \tau \in H[l]}>;
  T=sort(T);
  for(j=1; j<=c; j++) {
    if(H[j]==T) break;
  }
  if(j \in S) break;
  S=S \cup j;
  l=j;
}

```

5次対称群の部分群のうち、可移である以下の5個の共役類について、交換子群の列を示す。

H_{26} (位数5) の場合: $\{H_{26}, H_1$ (単位群) $\}$

H_{118} (位数10) の場合: $\{H_{118}, H_{30}$ (位数5, H_{26} の共役類), H_1 (単位群) $\}$

H_{120} (位数20) の場合: $\{H_{120}, H_{35}$ (位数5, H_{26} の共役類), H_1 (単位群) $\}$

H_{97} (位数60) の場合: $\{H_{97}\}$

H_{75} (位数120) の場合: $\{H_{75}, H_{97}$ (位数60) $\}$

これより、可移で可解な部分群(既約で可解な5次方程式のガロア群に対応する)は、 H_{26} (位数5), H_{118} (位数10), H_{120} (位数20) とそれらの共役類である。

4. 6次対称群の部分群

この場合は $n=6$, $N=720$ で, G は以下のように表される。(途中は省略)

$$\begin{aligned} G &= \{\sigma_1, \dots, \sigma_{121}, \dots, \sigma_{241}, \dots, \sigma_{361}, \dots, \sigma_{481}, \dots, \sigma_{601}, \dots, \sigma_{720}\} \\ &= \{\{1, 2, 3, 4, 5, 6\}, \dots, \{2, 1, 3, 4, 5, 6\}, \dots, \{3, 1, 2, 4, 5, 6\}, \dots, \\ &\quad \{4, 1, 2, 3, 5, 6\}, \dots, \{5, 1, 2, 3, 4, 6\}, \dots, \{6, 1, 2, 3, 4, 5\}, \dots, \{6, 5, 4, 3, 2, 1\}\} \end{aligned}$$

1個の元から生成される部分群は以下の362個である。ただし, 同一の共役類に属する部分群は, 代表の1個(最初に見つかったもの)だけを示す。

$$H_1 = \langle \sigma_1 \rangle = \{\sigma_1\} : \text{単位群, 位数1}$$

$$H_2 = \langle \sigma_2 \rangle = \{\sigma_1, \sigma_2\} : (5, 6) \text{ の置換, 位数2, 共役類は計5個}$$

$$H_4 = \langle \sigma_4 \rangle = \{\sigma_1, \sigma_4, \sigma_5\} : (4, 5, 6) \text{ の偶置換, 位数3, 共役類は計20個}$$

$$H_7 = \langle \sigma_8 \rangle = \{\sigma_1, \sigma_8\} : (3, 4) \text{ の置換と } (5, 6) \text{ の置換を同時に行ったもの, 位数2, 共役類は計45個}$$

$$H_9 = \langle \sigma_{10} \rangle = \{\sigma_1, \sigma_{10}, \sigma_{17}, \sigma_{19}\} : (3, 4, 5, 6) \text{ の巡回置換, 位数4, 共役類は計45個}$$

$$H_{21} = \langle \sigma_{28} \rangle = \{\sigma_1, \sigma_4, \sigma_5, \sigma_{25}, \sigma_{28}, \sigma_{29}\} : (2, 3) \text{ の置換と } (4, 5, 6) \text{ の偶置換の積, 位数6, 共役類は計60個}$$

$$H_{26} = \langle \sigma_{34} \rangle = \{\sigma_1, \sigma_{34}, \sigma_{65}, \sigma_{91}, \sigma_{97}\} : (2, 3, 4, 5, 6) \text{ の巡回置換, 位数5, 共役類は計36個}$$

$$H_{74} = \langle \sigma_{128} \rangle = \{\sigma_1, \sigma_{128}\} : (1, 2) \text{ の置換と } (3, 4) \text{ の置換と } (5, 6) \text{ の置換を同時に行ったもの, 位数2, 共役類は計15個}$$

$$H_{76} = \langle \sigma_{130} \rangle = \{\sigma_1, \sigma_{17}, \sigma_{130}, \sigma_{139}\} : (3, 4, 5, 6) \text{ の巡回置換で, そのうちの奇置換では } (1, 2) \text{ を交換したもの, 位数4, 共役類は計45個}$$

$$H_{88} = \langle \sigma_{148} \rangle = \{\sigma_1, \sigma_{148}, \sigma_{245}\} : (1, 2, 3) \text{ の偶置換と } (4, 5, 6) \text{ の偶置換を同時に行ったもの, 位数3, 共役類は計20個}$$

$$H_{94} = \langle \sigma_{154} \rangle = \{\sigma_1, \sigma_{154}, \sigma_{305}, \sigma_{451}, \sigma_{577}, \sigma_{601}\} : (1, 2, 3, 4, 5, 6) \text{ の巡回置換, 位数6, 共役類は計60個}$$

2個の元から生成される部分群は以下の1008個である。

$$H_{363} = \langle \sigma_2, \sigma_3 \rangle = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6\} : (4, 5, 6) \text{ の全置換, 位数6, 共役類は計20個}$$

$$H_{364} = \langle \sigma_2, \sigma_7 \rangle = \{\sigma_1, \sigma_2, \sigma_7, \sigma_8\} : (3, 4) \text{ の置換と } (5, 6) \text{ の置換の積, 位数4, 共役類は計45個}$$

$$H_{365} = \langle \sigma_2, \sigma_9 \rangle = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6, \sigma_7, \sigma_8, \sigma_9, \sigma_{10}, \sigma_{11}, \sigma_{12}, \sigma_{13}, \sigma_{14}, \sigma_{15}, \sigma_{16}, \sigma_{17}, \sigma_{18}, \sigma_{19}, \sigma_{20}, \sigma_{21}, \dots, \sigma_{24}\} : (3, 4, 5, 6) \text{ の全置換, 位数24, 共役類は計15個}$$

$$H_{367} = \langle \sigma_2, \sigma_{17} \rangle = \{\sigma_1, \sigma_2, \sigma_7, \sigma_8, \sigma_{17}, \sigma_{18}, \sigma_{23}, \sigma_{24}\} : (3, 5, 4, 6) \text{ の巡回置換と } (5, 6) \text{ の置換の積, 位数8, 共役類は計45個}$$

$$H_{369} = \langle \sigma_2, \sigma_{27} \rangle = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6, \sigma_{25}, \sigma_{26}, \sigma_{27}, \sigma_{28}, \sigma_{29}, \sigma_{30}\} : (2, 3) \text{ の置換と } (4, 5, 6) \text{ の全置換の積, 位数12, 共役類は計60個}$$

$$H_{370} = \langle \sigma_2, \sigma_{33} \rangle = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6, \sigma_7, \sigma_8, \sigma_9, \sigma_{10}, \sigma_{11}, \sigma_{12}, \sigma_{13}, \sigma_{14}, \sigma_{15}, \sigma_{16}, \sigma_{17}, \sigma_{18}, \sigma_{19}, \sigma_{20}, \dots, \sigma_{120}\} : (2, 3, 4, 5, 6) \text{ の全置換, 位数120, 共役類は計6個}$$

$$H_{381} = \langle \sigma_2, \sigma_{127} \rangle = \{\sigma_1, \sigma_2, \sigma_{127}, \sigma_{128}\} : (1, 2) \text{ の置換と } (3, 4) \text{ の置換を同時に行ったものと } (5, 6) \text{ の置換の積, 位数4, 共役類は計45個}$$

$$H_{382} = \langle \sigma_2, \sigma_{129} \rangle = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6, \sigma_7, \sigma_8, \sigma_9, \sigma_{10}, \sigma_{11}, \sigma_{12}, \sigma_{13}, \sigma_{14}, \sigma_{15}, \sigma_{16}, \sigma_{17}, \sigma_{18}, \sigma_{19}, \sigma_{20}, \dots, \sigma_{144}\} : (1, 2) \text{ の置換と } (3, 4, 5, 6) \text{ の全置換の積, 位数48, 共役類は計15個}$$

$$H_{384} = \langle \sigma_2, \sigma_{137} \rangle = \{\sigma_1, \sigma_2, \sigma_7, \sigma_8, \sigma_{137}, \sigma_{138}, \sigma_{143}, \sigma_{144}\} : (3, 4) \text{ の置換と } (5, 6) \text{ の置換の積と, これに } (1, 2) \text{ と } (3, 5) \text{ と } (4, 6) \text{ を3組同時に交換したものの和集合, 位数8, 共役類は計45個}$$

$$H_{385} = \langle \sigma_2, \sigma_{147} \rangle = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6, \sigma_{145}, \sigma_{146}, \sigma_{147}, \sigma_{148}, \sigma_{149}, \sigma_{150}, \sigma_{241}, \sigma_{242}, \sigma_{243}, \sigma_{244}, \sigma_{245}, \sigma_{246}\} : (1, 2, 3) \text{ の偶置換と } (4, 5, 6) \text{ の全置換の積, 位数18, 共役類は計20個}$$

$$H_{386} = \langle \sigma_2, \sigma_{151} \rangle = \{\sigma_1, \sigma_2, \sigma_{151}, \sigma_{152}, \sigma_{289}, \sigma_{290}, \sigma_{361}, \sigma_{362}\} : (1, 2, 3, 4) \text{ の巡回置換と } (5, 6) \text{ の置換の積, 位$$

数8, 共役類は計45個

$$H_{387} = \langle \sigma_2, \sigma_{153} \rangle = \{ \sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6, \sigma_7, \sigma_8, \sigma_9, \sigma_{10}, \sigma_{11}, \sigma_{12}, \sigma_{13}, \sigma_{14}, \sigma_{15}, \sigma_{16}, \sigma_{17}, \sigma_{18}, \sigma_{19}, \sigma_{20}, \dots, \sigma_{720} \} : \\ (1, 2, 3, 4, 5, 6) \text{の全置換, 位数}720$$

$$H_{389} = \langle \sigma_2, \sigma_{161} \rangle = \{ \sigma_1, \sigma_2, \sigma_7, \sigma_8, \sigma_{81}, \sigma_{82}, \sigma_{87}, \sigma_{88}, \sigma_{105}, \sigma_{106}, \sigma_{111}, \sigma_{112}, \sigma_{137}, \sigma_{138}, \sigma_{143}, \sigma_{144}, \sigma_{161}, \sigma_{162}, \dots, \sigma_{694} \} : \\ (1, 2, 3, 5, 4, 6) \text{の巡回置換と} (2, 5, 6) \text{の偶置換と} (3, 4) \text{の置換と} (5, 6) \text{の置換の積, 位数}72, \text{共役類は計}10 \text{個}$$

$$H_{396} = \langle \sigma_2, \sigma_{209} \rangle = \{ \sigma_1, \sigma_2, \sigma_{55}, \sigma_{56}, \sigma_{209}, \sigma_{210}, \sigma_{233}, \sigma_{234}, \sigma_{265}, \sigma_{266}, \sigma_{289}, \sigma_{290}, \sigma_{443}, \sigma_{444}, \sigma_{467}, \sigma_{468}, \sigma_{501}, \dots, \sigma_{668} \} : \\ (1, 2, 5, 3, 4, 6) \text{の巡回置換と} (2, 4) \text{の置換と} (5, 6) \text{の置換の積, 位数}24, \text{共役類は計}15 \text{個}$$

$$H_{483} = \langle \sigma_4, \sigma_8 \rangle = \{ \sigma_1, \sigma_4, \sigma_5, \sigma_8, \sigma_9, \sigma_{12}, \sigma_{13}, \sigma_{16}, \sigma_{17}, \sigma_{20}, \sigma_{21}, \sigma_{24} \} : (3, 4, 5, 6) \text{の偶置換, 位数}12, \text{共役類は計}15 \text{個}$$

$$H_{484} = \langle \sigma_4, \sigma_{26} \rangle = \{ \sigma_1, \sigma_4, \sigma_5, \sigma_{26}, \sigma_{27}, \sigma_{30} \} : (4, 5, 6) \text{の全置換で, そのうちの奇置換では} (2, 3) \text{を交換したものの, 位数}6, \text{共役類は計}60 \text{個}$$

$$H_{485} = \langle \sigma_4, \sigma_{31} \rangle = \{ \sigma_1, \sigma_4, \sigma_5, \sigma_8, \sigma_9, \sigma_{12}, \sigma_{13}, \sigma_{16}, \sigma_{17}, \sigma_{20}, \sigma_{21}, \sigma_{24}, \sigma_{26}, \sigma_{27}, \sigma_{30}, \sigma_{31}, \sigma_{34}, \sigma_{35}, \sigma_{38}, \sigma_{39}, \dots, \sigma_{120} \} : \\ (2, 3, 4, 5, 6) \text{の偶置換, 位数}60, \text{共役類は計}6 \text{個}$$

$$H_{488} = \langle \sigma_4, \sigma_{127} \rangle = \{ \sigma_1, \sigma_4, \sigma_5, \sigma_8, \sigma_9, \sigma_{12}, \sigma_{13}, \sigma_{16}, \sigma_{17}, \sigma_{20}, \sigma_{21}, \sigma_{24}, \sigma_{122}, \sigma_{123}, \sigma_{126}, \sigma_{127}, \sigma_{130}, \sigma_{131}, \dots, \sigma_{143} \} : \\ (3, 4, 5, 6) \text{の全置換で, そのうちの奇置換では} (1, 2) \text{を交換したものの, 位数}24, \text{共役類は計}15 \text{個}$$

$$H_{489} = \langle \sigma_4, \sigma_{128} \rangle = \{ \sigma_1, \sigma_4, \sigma_5, \sigma_8, \sigma_9, \sigma_{12}, \sigma_{13}, \sigma_{16}, \sigma_{17}, \sigma_{20}, \sigma_{21}, \sigma_{24}, \sigma_{121}, \sigma_{124}, \sigma_{125}, \sigma_{128}, \sigma_{129}, \sigma_{132}, \dots, \sigma_{144} \} : \\ (1, 2) \text{の置換と} (3, 4, 5, 6) \text{の偶置換の積, 位数}24, \text{共役類は計}15 \text{個}$$

$$H_{490} = \langle \sigma_4, \sigma_{145} \rangle = \{ \sigma_1, \sigma_4, \sigma_5, \sigma_{145}, \sigma_{148}, \sigma_{149}, \sigma_{241}, \sigma_{244}, \sigma_{245} \} : (1, 2, 3) \text{の偶置換と} (4, 5, 6) \text{の偶置換の積, 位数}9, \text{共役類は計}10 \text{個}$$

$$H_{491} = \langle \sigma_4, \sigma_{152} \rangle = \{ \sigma_1, \sigma_4, \sigma_5, \sigma_8, \sigma_9, \sigma_{12}, \sigma_{13}, \sigma_{16}, \sigma_{17}, \sigma_{20}, \sigma_{21}, \sigma_{24}, \sigma_{26}, \sigma_{27}, \sigma_{30}, \sigma_{31}, \sigma_{34}, \sigma_{35}, \sigma_{38}, \dots, \sigma_{719} \} : \\ (1, 2, 3, 4, 5, 6) \text{の偶置換, 位数}360$$

$$H_{500} = \langle \sigma_4, \sigma_{451} \rangle = \{ \sigma_1, \sigma_4, \sigma_5, \sigma_{145}, \sigma_{148}, \sigma_{149}, \sigma_{241}, \sigma_{244}, \sigma_{245}, \sigma_{451}, \sigma_{454}, \sigma_{455}, \sigma_{595}, \sigma_{598}, \sigma_{599}, \sigma_{691}, \sigma_{694}, \sigma_{695} \} : \\ (1, 2, 3) \text{の偶置換と} (4, 5, 6) \text{の偶置換の積と, これに} (1, 4) \text{と} (2, 5) \text{と} (3, 6) \text{を3組同時に交換したものの和集合, 位数}18, \text{共役類は計}20 \text{個}$$

$$H_{501} = \langle \sigma_4, \sigma_{452} \rangle = \{ \sigma_1, \sigma_4, \sigma_5, \sigma_{26}, \sigma_{27}, \sigma_{30}, \sigma_{122}, \sigma_{123}, \sigma_{126}, \sigma_{145}, \sigma_{148}, \sigma_{149}, \sigma_{241}, \sigma_{244}, \sigma_{245}, \sigma_{266}, \sigma_{267}, \dots, \sigma_{719} \} : \\ (1, 2, 3) \text{の全置換と} (4, 5, 6) \text{の全置換の積で, そのうちの奇置換では} (1, 4) \text{と} (2, 5) \text{と} (3, 6) \text{を3組同時に交換したものの, 位数}36, \text{共役類は計}10 \text{個}$$

$$H_{590} = \langle \sigma_8, \sigma_{17} \rangle = \{ \sigma_1, \sigma_8, \sigma_{17}, \sigma_{24} \} : \{ \{1, 2, 3, 4, 5, 6\}, \{1, 2, 4, 3, 6, 5\}, \{1, 2, 5, 6, 3, 4\}, \{1, 2, 6, 5, 4, 3\} \}, \\ \text{位数}4, \text{共役類は計}15 \text{個}$$

$$H_{592} = \langle \sigma_8, \sigma_{27} \rangle = \{ \sigma_1, \sigma_8, \sigma_{27}, \sigma_{38}, \sigma_{53}, \sigma_{68}, \sigma_{83}, \sigma_{94}, \sigma_{113}, \sigma_{120} \} : (3, 4) \text{の置換と} (5, 6) \text{の置換を同時に行つたものと} (2, 3, 5, 6, 4) \text{の巡回置換の積, 位数}10, \text{共役類は計}36 \text{個}$$

$$H_{594} = \langle \sigma_8, \sigma_{33} \rangle = \{ \sigma_1, \sigma_8, \sigma_{18}, \sigma_{23}, \sigma_{30}, \sigma_{33}, \sigma_{40}, \sigma_{43}, \sigma_{52}, \sigma_{59}, \sigma_{61}, \sigma_{70}, \sigma_{73}, \sigma_{80}, \sigma_{90}, \sigma_{95}, \sigma_{99}, \sigma_{108}, \sigma_{110}, \sigma_{117} \} : \\ (2, 3, 6, 5, 4) \text{の巡回置換と} (3, 5, 4, 6) \text{の巡回置換の積, 位数}20, \text{共役類は計}36 \text{個}$$

$$H_{598} = \langle \sigma_8, \sigma_{122} \rangle = \{ \sigma_1, \sigma_8, \sigma_{122}, \sigma_{127} \} : (1, 2) \text{の置換と} (3, 4) \text{の置換と} (5, 6) \text{の置換の積のうち, 偶置換だけを取り出したもの, 位数}4, \text{共役類は計}15 \text{個}$$

$$H_{599} = \langle \sigma_8, \sigma_{123} \rangle = \{ \sigma_1, \sigma_8, \sigma_{17}, \sigma_{24}, \sigma_{123}, \sigma_{131}, \sigma_{134}, \sigma_{142} \} : \{ \{1, 2, 3, 4, 5, 6\}, \{1, 2, 4, 3, 6, 5\}, \{1, 2, 5, 6, 3, 4\}, \{1, 2, 6, 5, 4, 3\}, \{2, 1, 3, 5, 4, 6\}, \{2, 1, 4, 6, 3, 5\}, \{2, 1, 5, 3, 6, 4\}, \{2, 1, 6, 4, 5, 3\} \}, \text{位数}8, \text{共役類は計}45 \text{個}$$

$$H_{601} = \langle \sigma_8, \sigma_{137} \rangle = \{ \sigma_1, \sigma_8, \sigma_{137}, \sigma_{144} \} : \{ \{1, 2, 3, 4, 5, 6\}, \{1, 2, 4, 3, 6, 5\}, \{2, 1, 5, 6, 3, 4\}, \{2, 1, 6, 5, 4, 3\} \}, \text{位数}4, \text{共役類は計}45 \text{個}$$

$$H_{603} = \langle \sigma_8, \sigma_{148} \rangle = \{ \sigma_1, \sigma_8, \sigma_{30}, \sigma_{43}, \sigma_{52}, \sigma_{61}, \sigma_{90}, \sigma_{95}, \sigma_{108}, \sigma_{117}, \sigma_{122}, \sigma_{127}, \sigma_{148}, \sigma_{157}, \sigma_{174}, \sigma_{187}, \sigma_{204}, \dots, \sigma_{703} \} : \\ (1, 2, 3, 4, 6, 5) \text{の巡回置換と} (2, 3, 6, 5, 4) \text{の巡回置換と} (3, 5, 4, 6) \text{の巡回置換の積のうち, 偶置換だけを取り出したもの, 位数}60, \text{共役類は計}6 \text{個}$$

- $H_{605} = \langle \sigma_8, \sigma_{154} \rangle = \{ \sigma_1, \sigma_8, \sigma_{18}, \sigma_{23}, \sigma_{27}, \sigma_{36}, \sigma_{38}, \sigma_{45}, \sigma_{53}, \sigma_{58}, \sigma_{63}, \sigma_{68}, \sigma_{76}, \sigma_{83}, \sigma_{85}, \sigma_{94}, \sigma_{98}, \sigma_{103}, \sigma_{113}, \dots, \sigma_{720} \} :$
 (1, 2, 3, 4, 5, 6)の巡回置換と(2, 3, 5, 6, 4)の巡回置換と(3, 5, 4, 6)の巡回置換の積, 位数120, 共役類は計6個
- $H_{608} = \langle \sigma_8, \sigma_{162} \rangle = \{ \sigma_1, \sigma_8, \sigma_{82}, \sigma_{87}, \sigma_{105}, \sigma_{112}, \sigma_{137}, \sigma_{144}, \sigma_{162}, \sigma_{167}, \sigma_{185}, \sigma_{192}, \sigma_{266}, \sigma_{271}, \sigma_{315}, \sigma_{326}, \sigma_{340}, \dots, \sigma_{694} \} :$
 $\{ \{1, 2, 3, 4, 5, 6\}, \{1, 2, 4, 3, 6, 5\}, \{2, 1, 5, 6, 3, 4\}, \{2, 1, 6, 5, 4, 3\} \}$ と(1, 3, 4)の偶置換と(2, 5, 6)の偶置換との積, 位数36, 共役類は計10個
- $H_{618} = \langle \sigma_8, \sigma_{290} \rangle = \{ \sigma_1, \sigma_8, \sigma_{122}, \sigma_{127}, \sigma_{290}, \sigma_{295}, \sigma_{409}, \sigma_{416} \} :$ (3, 4)の置換と(5, 6)の置換を同時に行ったものと(1, 3, 2, 4)の巡回置換の積, 位数8, 共役類は計45個
- $H_{619} = \langle \sigma_8, \sigma_{305} \rangle = \{ \sigma_1, \sigma_8, \sigma_{122}, \sigma_{127}, \sigma_{305}, \sigma_{312}, \sigma_{426}, \sigma_{431}, \sigma_{577}, \sigma_{584}, \sigma_{698}, \sigma_{703} \} :$ (1, 3, 5, 2, 4, 6)の巡回置換と(3, 4)の置換と(5, 6)の置換の積のうち, 偶置換だけを取り出したもの, 位数12, 共役類は計15個
- $H_{620} = \langle \sigma_8, \sigma_{315} \rangle = \{ \sigma_1, \sigma_8, \sigma_{315}, \sigma_{326}, \sigma_{459}, \sigma_{470} \} :$ (1, 3, 4)の全置換と(2, 5, 6)の全置換を同時に行ったもの, 位数6, 共役類は計60個
- $H_{621} = \langle \sigma_8, \sigma_{316} \rangle = \{ \sigma_1, \sigma_2, \sigma_7, \sigma_8, \sigma_{81}, \sigma_{82}, \sigma_{87}, \sigma_{88}, \sigma_{105}, \sigma_{106}, \sigma_{111}, \sigma_{112}, \sigma_{265}, \sigma_{266}, \sigma_{271}, \sigma_{272}, \sigma_{315}, \sigma_{316}, \dots, \sigma_{470} \} :$
 (1, 3, 4)の全置換と(2, 5, 6)の全置換の積, 位数36, 共役類は計10個
- $H_{622} = \langle \sigma_8, \sigma_{317} \rangle = \{ \sigma_1, \sigma_8, \sigma_{137}, \sigma_{144}, \sigma_{317}, \sigma_{332}, \sigma_{461}, \sigma_{476}, \sigma_{537}, \sigma_{544}, \sigma_{681}, \sigma_{688} \} :$ (3, 4)の置換と(5, 6)の置換を同時に行ったものと(1, 3, 6, 2, 5, 4)の巡回置換の積, 位数12, 共役類は計60個
- $H_{623} = \langle \sigma_8, \sigma_{321} \rangle = \{ \sigma_1, \sigma_6, \sigma_8, \sigma_{10}, \sigma_{15}, \sigma_{17}, \sigma_{19}, \sigma_{24}, \sigma_{121}, \sigma_{126}, \sigma_{128}, \sigma_{130}, \sigma_{135}, \sigma_{137}, \sigma_{139}, \sigma_{144}, \sigma_{315}, \dots, \sigma_{694} \} :$
 (1, 3, 4, 2, 5, 6)の巡回置換と(3, 4, 5, 6)の巡回置換と(4, 6)の置換の積, 位数48, 共役類は計15個
- $H_{624} = \langle \sigma_8, \sigma_{323} \rangle = \{ \sigma_1, \sigma_8, \sigma_{17}, \sigma_{24}, \sigma_{126}, \sigma_{130}, \sigma_{135}, \sigma_{139}, \sigma_{315}, \sigma_{323}, \sigma_{326}, \sigma_{334}, \sigma_{459}, \sigma_{467}, \sigma_{470}, \sigma_{478}, \sigma_{533}, \dots, \sigma_{692} \} :$
 (1, 3, 4, 2, 5, 6)の巡回置換と(3, 4, 5, 6)の巡回置換と(4, 6)の置換の積のうち, 偶置換だけを取り出したもの, 位数24, 共役類は計15個
- $H_{659} = \langle \sigma_{10}, \sigma_{317} \rangle = \{ \sigma_1, \sigma_{10}, \sigma_{17}, \sigma_{19}, \sigma_{126}, \sigma_{128}, \sigma_{135}, \sigma_{144}, \sigma_{317}, \sigma_{321}, \sigma_{326}, \sigma_{334}, \sigma_{459}, \sigma_{467}, \sigma_{472}, \sigma_{476}, \dots, \sigma_{694} \} :$
 (1, 3, 6)の偶置換と(2, 5, 4)の偶置換を同時に行ったものと(3, 4, 5, 6)の巡回置換の積と, これに(1, 2)と(3, 4)と(5, 6)を3組同時に交換したものの和集合, 位数24, 共役類は計15個
- $H_{1321} = \langle \sigma_{128}, \sigma_{317} \rangle = \{ \sigma_1, \sigma_{128}, \sigma_{317}, \sigma_{467}, \sigma_{548}, \sigma_{694} \} :$ (1, 4, 5)の偶置換と(2, 6, 3)の偶置換を同時に行ったものと, これに(1, 2)と(3, 4)と(5, 6)を3組同時に交換したものの和集合, 位数6, 共役類は計20個

3個の元から生成される部分群は以下の85個である。

- $H_{1371} = \langle \sigma_2, \sigma_7, \sigma_{121} \rangle = \{ \sigma_1, \sigma_2, \sigma_7, \sigma_8, \sigma_{121}, \sigma_{122}, \sigma_{127}, \sigma_{128} \} :$ (1, 2)の置換と(3, 4)の置換と(5, 6)の置換の積, 位数8, 共役類は計15個
- $H_{1372} = \langle \sigma_2, \sigma_7, \sigma_{289} \rangle = \{ \sigma_1, \sigma_2, \sigma_7, \sigma_8, \sigma_{121}, \sigma_{122}, \sigma_{127}, \sigma_{128}, \sigma_{289}, \sigma_{290}, \sigma_{295}, \sigma_{296}, \sigma_{409}, \sigma_{410}, \sigma_{415}, \sigma_{416} \} :$ (1, 3, 2, 4)の巡回置換と(3, 4)の置換と(5, 6)の置換の積, 位数16, 共役類は計45個
- $H_{1383} = \langle \sigma_2, \sigma_{127}, \sigma_{289} \rangle = \{ \sigma_1, \sigma_2, \sigma_{127}, \sigma_{128}, \sigma_{289}, \sigma_{290}, \sigma_{415}, \sigma_{416} \} :$ $\{ \{1, 2, 3, 4, 5, 6\}, \{2, 1, 4, 3, 5, 6\}, \{3, 4, 1, 2, 5, 6\}, \{4, 3, 2, 1, 5, 6\} \}$ と(5, 6)の置換の積, 位数8, 共役類は計15個
- $H_{1397} = \langle \sigma_4, \sigma_{26}, \sigma_{122} \rangle = \{ \sigma_1, \sigma_4, \sigma_5, \sigma_{26}, \sigma_{27}, \sigma_{30}, \sigma_{122}, \sigma_{123}, \sigma_{126}, \sigma_{145}, \sigma_{148}, \sigma_{149}, \sigma_{241}, \sigma_{244}, \sigma_{245}, \sigma_{266}, \sigma_{267}, \sigma_{270} \} :$
 (1, 2, 3)の全置換と(4, 5, 6)の全置換の積のうち, 偶置換だけを取り出したもの, 位数18, 共役類は計10個

4個の元から生成される部分群はない。以上より6次対称群の部分群は1455個である。6次対称群の部分群を共役類で分類して下表に示す。この結果は参考文献(2)の4-5頁の表と一致する。

位数	部分群
1	$H_1 = \langle \sigma_1 \rangle$
2	$H_2 = \langle \sigma_2 \rangle, H_3 = \langle \sigma_3 \rangle, H_5 = \langle \sigma_6 \rangle, H_6 = \langle \sigma_7 \rangle, H_{12} = \langle \sigma_{15} \rangle, H_{16} = \langle \sigma_{22} \rangle, H_{18} = \langle \sigma_{25} \rangle, H_{41} = \langle \sigma_{55} \rangle, H_{54} = \langle \sigma_{81} \rangle,$ $H_{63} = \langle \sigma_{106} \rangle, H_{68} = \langle \sigma_{121} \rangle, H_{181} = \langle \sigma_{265} \rangle, H_{258} = \langle \sigma_{391} \rangle, H_{311} = \langle \sigma_{513} \rangle, H_{346} = \langle \sigma_{634} \rangle$ $H_7 = \langle \sigma_8 \rangle, H_{14} = \langle \sigma_{17} \rangle, H_{17} = \langle \sigma_{24} \rangle, H_{19} = \langle \sigma_{26} \rangle, H_{20} = \langle \sigma_{27} \rangle, H_{22} = \langle \sigma_{30} \rangle, H_{42} = \langle \sigma_{56} \rangle, H_{47} = \langle \sigma_{61} \rangle, H_{51} = \langle \sigma_{68} \rangle,$ $H_{56} = \langle \sigma_{83} \rangle, H_{58} = \langle \sigma_{87} \rangle, H_{61} = \langle \sigma_{95} \rangle, H_{64} = \langle \sigma_{108} \rangle, H_{65} = \langle \sigma_{112} \rangle, H_{67} = \langle \sigma_{120} \rangle, H_{69} = \langle \sigma_{122} \rangle, H_{70} = \langle \sigma_{123} \rangle, H_{72} = \langle \sigma_{126} \rangle,$ $H_{73} = \langle \sigma_{127} \rangle, H_{79} = \langle \sigma_{135} \rangle, H_{83} = \langle \sigma_{142} \rangle, H_{182} = \langle \sigma_{266} \rangle, H_{183} = \langle \sigma_{267} \rangle, H_{185} = \langle \sigma_{270} \rangle, H_{204} = \langle \sigma_{289} \rangle, H_{224} = \langle \sigma_{315} \rangle,$ $H_{242} = \langle \sigma_{340} \rangle, H_{259} = \langle \sigma_{392} \rangle, H_{264} = \langle \sigma_{397} \rangle, H_{268} = \langle \sigma_{404} \rangle, H_{271} = \langle \sigma_{415} \rangle, H_{290} = \langle \sigma_{445} \rangle, H_{303} = \langle \sigma_{470} \rangle, H_{313} = \langle \sigma_{515} \rangle,$ $H_{315} = \langle \sigma_{519} \rangle, H_{318} = \langle \sigma_{527} \rangle, H_{320} = \langle \sigma_{537} \rangle, H_{333} = \langle \sigma_{567} \rangle, H_{341} = \langle \sigma_{593} \rangle, H_{347} = \langle \sigma_{636} \rangle, H_{348} = \langle \sigma_{640} \rangle, H_{350} = \langle \sigma_{648} \rangle,$ $H_{351} = \langle \sigma_{658} \rangle, H_{357} = \langle \sigma_{688} \rangle, H_{361} = \langle \sigma_{714} \rangle$ $H_{74} = \langle \sigma_{128} \rangle, H_{81} = \langle \sigma_{137} \rangle, H_{84} = \langle \sigma_{144} \rangle, H_{205} = \langle \sigma_{290} \rangle, H_{226} = \langle \sigma_{317} \rangle, H_{243} = \langle \sigma_{342} \rangle, H_{272} = \langle \sigma_{416} \rangle, H_{294} = \langle \sigma_{451} \rangle,$ $H_{306} = \langle \sigma_{476} \rangle, H_{322} = \langle \sigma_{539} \rangle, H_{336} = \langle \sigma_{573} \rangle, H_{344} = \langle \sigma_{599} \rangle, H_{352} = \langle \sigma_{660} \rangle, H_{359} = \langle \sigma_{694} \rangle, H_{362} = \langle \sigma_{720} \rangle$
3	$H_4 = \langle \sigma_4 \rangle, H_8 = \langle \sigma_9 \rangle, H_{11} = \langle \sigma_{12} \rangle, H_{13} = \langle \sigma_{16} \rangle, H_{23} = \langle \sigma_{31} \rangle, H_{31} = \langle \sigma_{39} \rangle, H_{38} = \langle \sigma_{46} \rangle, H_{43} = \langle \sigma_{57} \rangle, H_{46} = \langle \sigma_{60} \rangle,$ $H_{55} = \langle \sigma_{82} \rangle, H_{85} = \langle \sigma_{145} \rangle, H_{115} = \langle \sigma_{175} \rangle, H_{141} = \langle \sigma_{201} \rangle, H_{166} = \langle \sigma_{226} \rangle, H_{186} = \langle \sigma_{271} \rangle, H_{194} = \langle \sigma_{279} \rangle, H_{201} = \langle \sigma_{286} \rangle,$ $H_{260} = \langle \sigma_{393} \rangle, H_{263} = \langle \sigma_{396} \rangle, H_{312} = \langle \sigma_{514} \rangle$ $H_{88} = \langle \sigma_{148} \rangle, H_{89} = \langle \sigma_{149} \rangle, H_{122} = \langle \sigma_{182} \rangle, H_{127} = \langle \sigma_{187} \rangle, H_{149} = \langle \sigma_{209} \rangle, H_{153} = \langle \sigma_{213} \rangle, H_{174} = \langle \sigma_{234} \rangle, H_{178} = \langle \sigma_{238} \rangle,$ $H_{217} = \langle \sigma_{305} \rangle, H_{223} = \langle \sigma_{312} \rangle, H_{233} = \langle \sigma_{326} \rangle, H_{240} = \langle \sigma_{334} \rangle, H_{248} = \langle \sigma_{349} \rangle, H_{255} = \langle \sigma_{357} \rangle, H_{281} = \langle \sigma_{426} \rangle, H_{285} = \langle \sigma_{431} \rangle,$ $H_{289} = \langle \sigma_{444} \rangle, H_{301} = \langle \sigma_{467} \rangle, H_{325} = \langle \sigma_{544} \rangle, H_{331} = \langle \sigma_{562} \rangle$
4	$H_9 = \langle \sigma_{10} \rangle, H_{10} = \langle \sigma_{11} \rangle, H_{15} = \langle \sigma_{18} \rangle, H_{25} = \langle \sigma_{33} \rangle, H_{28} = \langle \sigma_{36} \rangle, H_{29} = \langle \sigma_{37} \rangle, H_{32} = \langle \sigma_{40} \rangle, H_{36} = \langle \sigma_{44} \rangle, H_{37} = \langle \sigma_{45} \rangle,$ $H_{44} = \langle \sigma_{58} \rangle, H_{45} = \langle \sigma_{59} \rangle, H_{49} = \langle \sigma_{63} \rangle, H_{52} = \langle \sigma_{70} \rangle, H_{57} = \langle \sigma_{84} \rangle, H_{62} = \langle \sigma_{96} \rangle, H_{91} = \langle \sigma_{151} \rangle, H_{99} = \langle \sigma_{159} \rangle, H_{106} = \langle \sigma_{166} \rangle,$ $H_{109} = \langle \sigma_{169} \rangle, H_{117} = \langle \sigma_{177} \rangle, H_{120} = \langle \sigma_{180} \rangle, H_{135} = \langle \sigma_{195} \rangle, H_{139} = \langle \sigma_{199} \rangle, H_{142} = \langle \sigma_{202} \rangle, H_{160} = \langle \sigma_{220} \rangle, H_{164} = \langle \sigma_{224} \rangle,$ $H_{165} = \langle \sigma_{225} \rangle, H_{188} = \langle \sigma_{273} \rangle, H_{191} = \langle \sigma_{276} \rangle, H_{192} = \langle \sigma_{277} \rangle, H_{195} = \langle \sigma_{280} \rangle, H_{199} = \langle \sigma_{284} \rangle, H_{200} = \langle \sigma_{285} \rangle, H_{210} = \langle \sigma_{295} \rangle,$ $H_{229} = \langle \sigma_{321} \rangle, H_{245} = \langle \sigma_{346} \rangle, H_{261} = \langle \sigma_{394} \rangle, H_{262} = \langle \sigma_{395} \rangle, H_{266} = \langle \sigma_{399} \rangle, H_{269} = \langle \sigma_{406} \rangle, H_{292} = \langle \sigma_{447} \rangle, H_{304} = \langle \sigma_{472} \rangle,$ $H_{314} = \langle \sigma_{516} \rangle, H_{319} = \langle \sigma_{528} \rangle, H_{342} = \langle \sigma_{594} \rangle$ $H_{76} = \langle \sigma_{130} \rangle, H_{77} = \langle \sigma_{131} \rangle, H_{82} = \langle \sigma_{138} \rangle, H_{92} = \langle \sigma_{152} \rangle, H_{101} = \langle \sigma_{161} \rangle, H_{108} = \langle \sigma_{168} \rangle, H_{110} = \langle \sigma_{170} \rangle, H_{126} = \langle \sigma_{186} \rangle,$ $H_{131} = \langle \sigma_{191} \rangle, H_{137} = \langle \sigma_{197} \rangle, H_{148} = \langle \sigma_{208} \rangle, H_{152} = \langle \sigma_{212} \rangle, H_{162} = \langle \sigma_{222} \rangle, H_{171} = \langle \sigma_{231} \rangle, H_{175} = \langle \sigma_{235} \rangle, H_{207} = \langle \sigma_{292} \rangle,$ $H_{208} = \langle \sigma_{293} \rangle, H_{211} = \langle \sigma_{296} \rangle, H_{216} = \langle \sigma_{304} \rangle, H_{220} = \langle \sigma_{309} \rangle, H_{227} = \langle \sigma_{318} \rangle, H_{230} = \langle \sigma_{323} \rangle, H_{236} = \langle \sigma_{330} \rangle, H_{237} = \langle \sigma_{331} \rangle,$ $H_{247} = \langle \sigma_{348} \rangle, H_{251} = \langle \sigma_{353} \rangle, H_{254} = \langle \sigma_{356} \rangle, H_{274} = \langle \sigma_{418} \rangle, H_{275} = \langle \sigma_{419} \rangle, H_{278} = \langle \sigma_{422} \rangle, H_{282} = \langle \sigma_{427} \rangle, H_{295} = \langle \sigma_{452} \rangle,$ $H_{296} = \langle \sigma_{453} \rangle, H_{299} = \langle \sigma_{456} \rangle, H_{308} = \langle \sigma_{478} \rangle, H_{309} = \langle \sigma_{479} \rangle, H_{323} = \langle \sigma_{540} \rangle, H_{326} = \langle \sigma_{545} \rangle, H_{328} = \langle \sigma_{549} \rangle, H_{337} = \langle \sigma_{574} \rangle,$ $H_{338} = \langle \sigma_{575} \rangle, H_{345} = \langle \sigma_{600} \rangle, H_{354} = \langle \sigma_{666} \rangle, H_{355} = \langle \sigma_{670} \rangle, H_{360} = \langle \sigma_{696} \rangle$ $H_{364} = \langle \sigma_2, \sigma_7 \rangle, H_{368} = \langle \sigma_2, \sigma_{25} \rangle, H_{372} = \langle \sigma_2, \sigma_{55} \rangle, H_{379} = \langle \sigma_2, \sigma_{121} \rangle, H_{398} = \langle \sigma_2, \sigma_{265} \rangle, H_{411} = \langle \sigma_2, \sigma_{391} \rangle,$ $H_{433} = \langle \sigma_3, \sigma_{22} \rangle, H_{434} = \langle \sigma_3, \sigma_{25} \rangle, H_{440} = \langle \sigma_3, \sigma_{106} \rangle, H_{442} = \langle \sigma_3, \sigma_{121} \rangle, H_{456} = \langle \sigma_3, \sigma_{265} \rangle, H_{477} = \langle \sigma_3, \sigma_{634} \rangle,$ $H_{505} = \langle \sigma_6, \sigma_{15} \rangle, H_{506} = \langle \sigma_6, \sigma_{25} \rangle, H_{512} = \langle \sigma_6, \sigma_{81} \rangle, H_{514} = \langle \sigma_6, \sigma_{121} \rangle, H_{528} = \langle \sigma_6, \sigma_{265} \rangle, H_{549} = \langle \sigma_6, \sigma_{513} \rangle,$ $H_{559} = \langle \sigma_7, \sigma_{81} \rangle, H_{560} = \langle \sigma_7, \sigma_{106} \rangle, H_{561} = \langle \sigma_7, \sigma_{121} \rangle, H_{580} = \langle \sigma_7, \sigma_{513} \rangle, H_{586} = \langle \sigma_7, \sigma_{634} \rangle, H_{687} = \langle \sigma_{15}, \sigma_{55} \rangle,$ $H_{688} = \langle \sigma_{15}, \sigma_{106} \rangle, H_{689} = \langle \sigma_{15}, \sigma_{121} \rangle, H_{708} = \langle \sigma_{15}, \sigma_{391} \rangle, H_{714} = \langle \sigma_{15}, \sigma_{634} \rangle, H_{776} = \langle \sigma_{22}, \sigma_{55} \rangle, H_{777} = \langle \sigma_{22}, \sigma_{81} \rangle,$ $H_{778} = \langle \sigma_{22}, \sigma_{121} \rangle, H_{797} = \langle \sigma_{22}, \sigma_{391} \rangle, H_{803} = \langle \sigma_{22}, \sigma_{513} \rangle, H_{852} = \langle \sigma_{25}, \sigma_{391} \rangle, H_{856} = \langle \sigma_{25}, \sigma_{513} \rangle, H_{859} = \langle \sigma_{25}, \sigma_{634} \rangle,$ $H_{1012} = \langle \sigma_{55}, \sigma_{265} \rangle, H_{1016} = \langle \sigma_{55}, \sigma_{513} \rangle, H_{1019} = \langle \sigma_{55}, \sigma_{634} \rangle, H_{1139} = \langle \sigma_{81}, \sigma_{265} \rangle, H_{1143} = \langle \sigma_{81}, \sigma_{391} \rangle, H_{1146} = \langle \sigma_{81}, \sigma_{634} \rangle,$ $H_{1240} = \langle \sigma_{106}, \sigma_{265} \rangle, H_{1244} = \langle \sigma_{106}, \sigma_{391} \rangle, H_{1247} = \langle \sigma_{106}, \sigma_{513} \rangle$ $H_{381} = \langle \sigma_2, \sigma_{127} \rangle, H_{402} = \langle \sigma_2, \sigma_{289} \rangle, H_{414} = \langle \sigma_2, \sigma_{415} \rangle, H_{445} = \langle \sigma_3, \sigma_{142} \rangle, H_{462} = \langle \sigma_3, \sigma_{340} \rangle, H_{479} = \langle \sigma_3, \sigma_{658} \rangle,$ $H_{517} = \langle \sigma_6, \sigma_{135} \rangle, H_{534} = \langle \sigma_6, \sigma_{315} \rangle, H_{551} = \langle \sigma_6, \sigma_{537} \rangle, H_{562} = \langle \sigma_7, \sigma_{122} \rangle, H_{584} = \langle \sigma_7, \sigma_{593} \rangle, H_{589} = \langle \sigma_7, \sigma_{714} \rangle,$ $H_{597} = \langle \sigma_8, \sigma_{121} \rangle, H_{690} = \langle \sigma_{15}, \sigma_{126} \rangle, H_{712} = \langle \sigma_{15}, \sigma_{470} \rangle, H_{717} = \langle \sigma_{15}, \sigma_{688} \rangle, H_{736} = \langle \sigma_{17}, \sigma_{121} \rangle, H_{779} = \langle \sigma_{22}, \sigma_{123} \rangle,$ $H_{801} = \langle \sigma_{22}, \sigma_{445} \rangle, H_{806} = \langle \sigma_{22}, \sigma_{567} \rangle, H_{811} = \langle \sigma_{24}, \sigma_{121} \rangle, H_{853} = \langle \sigma_{25}, \sigma_{392} \rangle, H_{857} = \langle \sigma_{25}, \sigma_{515} \rangle, H_{860} = \langle \sigma_{25}, \sigma_{636} \rangle,$ $H_{879} = \langle \sigma_{26}, \sigma_{391} \rangle, H_{916} = \langle \sigma_{27}, \sigma_{634} \rangle, H_{945} = \langle \sigma_{30}, \sigma_{513} \rangle, H_{1013} = \langle \sigma_{55}, \sigma_{266} \rangle, H_{1017} = \langle \sigma_{55}, \sigma_{527} \rangle, H_{1020} = \langle \sigma_{55}, \sigma_{648} \rangle,$ $H_{1039} = \langle \sigma_{56}, \sigma_{265} \rangle, H_{1095} = \langle \sigma_{61}, \sigma_{634} \rangle, H_{1121} = \langle \sigma_{68}, \sigma_{513} \rangle, H_{1140} = \langle \sigma_{81}, \sigma_{270} \rangle, H_{1144} = \langle \sigma_{81}, \sigma_{404} \rangle, H_{1147} = \langle \sigma_{81}, \sigma_{640} \rangle,$ $H_{1168} = \langle \sigma_{83}, \sigma_{265} \rangle, H_{1200} = \langle \sigma_{87}, \sigma_{634} \rangle, H_{1222} = \langle \sigma_{95}, \sigma_{391} \rangle, H_{1241} = \langle \sigma_{106}, \sigma_{267} \rangle, H_{1245} = \langle \sigma_{106}, \sigma_{397} \rangle,$ $H_{1248} = \langle \sigma_{106}, \sigma_{519} \rangle, H_{1260} = \langle \sigma_{108}, \sigma_{265} \rangle, H_{1285} = \langle \sigma_{112}, \sigma_{513} \rangle, H_{1308} = \langle \sigma_{120}, \sigma_{391} \rangle$

位数	部分群
4	$H_{590}=\langle\sigma_8, \sigma_{17}\rangle, H_{861}=\langle\sigma_{26}, \sigma_{95}\rangle, H_{892}=\langle\sigma_{27}, \sigma_{61}\rangle, H_{921}=\langle\sigma_{30}, \sigma_{68}\rangle, H_{1021}=\langle\sigma_{56}, \sigma_{83}\rangle, H_{1311}=\langle\sigma_{122}, \sigma_{593}\rangle,$ $H_{1313}=\langle\sigma_{123}, \sigma_{445}\rangle, H_{1315}=\langle\sigma_{126}, \sigma_{470}\rangle, H_{1317}=\langle\sigma_{127}, \sigma_{289}\rangle, H_{1327}=\langle\sigma_{135}, \sigma_{315}\rangle, H_{1337}=\langle\sigma_{142}, \sigma_{340}\rangle,$ $H_{1355}=\langle\sigma_{266}, \sigma_{527}\rangle, H_{1357}=\langle\sigma_{267}, \sigma_{397}\rangle, H_{1359}=\langle\sigma_{270}, \sigma_{404}\rangle, H_{1367}=\langle\sigma_{392}, \sigma_{515}\rangle$
	$H_{598}=\langle\sigma_8, \sigma_{122}\rangle, H_{738}=\langle\sigma_{17}, \sigma_{126}\rangle, H_{812}=\langle\sigma_{24}, \sigma_{123}\rangle, H_{880}=\langle\sigma_{26}, \sigma_{392}\rangle, H_{917}=\langle\sigma_{27}, \sigma_{636}\rangle, H_{946}=\langle\sigma_{30}, \sigma_{515}\rangle,$ $H_{1040}=\langle\sigma_{56}, \sigma_{266}\rangle, H_{1097}=\langle\sigma_{61}, \sigma_{648}\rangle, H_{1123}=\langle\sigma_{68}, \sigma_{527}\rangle, H_{1170}=\langle\sigma_{83}, \sigma_{270}\rangle, H_{1201}=\langle\sigma_{87}, \sigma_{640}\rangle, H_{1224}=\langle\sigma_{95}, \sigma_{404}\rangle,$ $H_{1261}=\langle\sigma_{108}, \sigma_{267}\rangle, H_{1289}=\langle\sigma_{112}, \sigma_{519}\rangle, H_{1309}=\langle\sigma_{120}, \sigma_{397}\rangle$
	$H_{601}=\langle\sigma_8, \sigma_{137}\rangle, H_{739}=\langle\sigma_{17}, \sigma_{128}\rangle, H_{813}=\langle\sigma_{24}, \sigma_{128}\rangle, H_{883}=\langle\sigma_{26}, \sigma_{451}\rangle, H_{920}=\langle\sigma_{27}, \sigma_{694}\rangle, H_{949}=\langle\sigma_{30}, \sigma_{573}\rangle,$ $H_{1043}=\langle\sigma_{56}, \sigma_{317}\rangle, H_{1098}=\langle\sigma_{61}, \sigma_{660}\rangle, H_{1124}=\langle\sigma_{68}, \sigma_{539}\rangle, H_{1171}=\langle\sigma_{83}, \sigma_{290}\rangle, H_{1202}=\langle\sigma_{87}, \sigma_{660}\rangle, H_{1225}=\langle\sigma_{95}, \sigma_{416}\rangle,$ $H_{1262}=\langle\sigma_{108}, \sigma_{290}\rangle, H_{1290}=\langle\sigma_{112}, \sigma_{539}\rangle, H_{1310}=\langle\sigma_{120}, \sigma_{416}\rangle, H_{1312}=\langle\sigma_{122}, \sigma_{599}\rangle, H_{1314}=\langle\sigma_{123}, \sigma_{451}\rangle,$ $H_{1316}=\langle\sigma_{126}, \sigma_{476}\rangle, H_{1318}=\langle\sigma_{127}, \sigma_{290}\rangle, H_{1319}=\langle\sigma_{128}, \sigma_{289}\rangle, H_{1320}=\langle\sigma_{128}, \sigma_{290}\rangle, H_{1325}=\langle\sigma_{128}, \sigma_{593}\rangle,$ $H_{1326}=\langle\sigma_{128}, \sigma_{599}\rangle, H_{1328}=\langle\sigma_{135}, \sigma_{317}\rangle, H_{1331}=\langle\sigma_{137}, \sigma_{315}\rangle, H_{1332}=\langle\sigma_{137}, \sigma_{317}\rangle, H_{1335}=\langle\sigma_{137}, \sigma_{470}\rangle,$ $H_{1336}=\langle\sigma_{137}, \sigma_{476}\rangle, H_{1338}=\langle\sigma_{142}, \sigma_{342}\rangle, H_{1343}=\langle\sigma_{144}, \sigma_{340}\rangle, H_{1344}=\langle\sigma_{144}, \sigma_{342}\rangle, H_{1345}=\langle\sigma_{144}, \sigma_{445}\rangle,$ $H_{1346}=\langle\sigma_{144}, \sigma_{451}\rangle, H_{1356}=\langle\sigma_{266}, \sigma_{573}\rangle, H_{1358}=\langle\sigma_{267}, \sigma_{476}\rangle, H_{1360}=\langle\sigma_{270}, \sigma_{451}\rangle, H_{1361}=\langle\sigma_{290}, \sigma_{527}\rangle,$ $H_{1362}=\langle\sigma_{290}, \sigma_{573}\rangle, H_{1363}=\langle\sigma_{317}, \sigma_{404}\rangle, H_{1364}=\langle\sigma_{317}, \sigma_{451}\rangle, H_{1365}=\langle\sigma_{342}, \sigma_{397}\rangle, H_{1366}=\langle\sigma_{342}, \sigma_{476}\rangle,$ $H_{1368}=\langle\sigma_{392}, \sigma_{539}\rangle, H_{1369}=\langle\sigma_{416}, \sigma_{515}\rangle, H_{1370}=\langle\sigma_{416}, \sigma_{539}\rangle$
5	$H_{26}=\langle\sigma_{34}\rangle, H_{27}=\langle\sigma_{35}\rangle, H_{30}=\langle\sigma_{38}\rangle, H_{34}=\langle\sigma_{42}\rangle, H_{35}=\langle\sigma_{43}\rangle, H_{39}=\langle\sigma_{47}\rangle, H_{93}=\langle\sigma_{153}\rangle, H_{96}=\langle\sigma_{156}\rangle, H_{97}=\langle\sigma_{157}\rangle,$ $H_{100}=\langle\sigma_{160}\rangle, H_{104}=\langle\sigma_{164}\rangle, H_{105}=\langle\sigma_{165}\rangle, H_{111}=\langle\sigma_{171}\rangle, H_{114}=\langle\sigma_{174}\rangle, H_{118}=\langle\sigma_{178}\rangle, H_{119}=\langle\sigma_{179}\rangle, H_{123}=\langle\sigma_{183}\rangle,$ $H_{130}=\langle\sigma_{190}\rangle, H_{133}=\langle\sigma_{193}\rangle, H_{136}=\langle\sigma_{196}\rangle, H_{140}=\langle\sigma_{200}\rangle, H_{144}=\langle\sigma_{204}\rangle, H_{145}=\langle\sigma_{205}\rangle, H_{156}=\langle\sigma_{216}\rangle, H_{158}=\langle\sigma_{218}\rangle,$ $H_{159}=\langle\sigma_{219}\rangle, H_{163}=\langle\sigma_{223}\rangle, H_{167}=\langle\sigma_{227}\rangle, H_{170}=\langle\sigma_{230}\rangle, H_{179}=\langle\sigma_{239}\rangle, H_{189}=\langle\sigma_{274}\rangle, H_{190}=\langle\sigma_{275}\rangle, H_{193}=\langle\sigma_{278}\rangle,$ $H_{197}=\langle\sigma_{282}\rangle, H_{198}=\langle\sigma_{283}\rangle, H_{202}=\langle\sigma_{287}\rangle$
6	$H_{21}=\langle\sigma_{28}\rangle, H_{24}=\langle\sigma_{32}\rangle, H_{33}=\langle\sigma_{41}\rangle, H_{40}=\langle\sigma_{48}\rangle, H_{48}=\langle\sigma_{62}\rangle, H_{50}=\langle\sigma_{66}\rangle, H_{53}=\langle\sigma_{71}\rangle, H_{59}=\langle\sigma_{88}\rangle, H_{60}=\langle\sigma_{89}\rangle,$ $H_{66}=\langle\sigma_{114}\rangle, H_{71}=\langle\sigma_{124}\rangle, H_{75}=\langle\sigma_{129}\rangle, H_{78}=\langle\sigma_{132}\rangle, H_{80}=\langle\sigma_{136}\rangle, H_{86}=\langle\sigma_{146}\rangle, H_{87}=\langle\sigma_{147}\rangle, H_{90}=\langle\sigma_{150}\rangle,$ $H_{116}=\langle\sigma_{176}\rangle, H_{121}=\langle\sigma_{181}\rangle, H_{128}=\langle\sigma_{188}\rangle, H_{143}=\langle\sigma_{203}\rangle, H_{147}=\langle\sigma_{207}\rangle, H_{155}=\langle\sigma_{215}\rangle, H_{168}=\langle\sigma_{228}\rangle, H_{172}=\langle\sigma_{232}\rangle,$ $H_{180}=\langle\sigma_{240}\rangle, H_{184}=\langle\sigma_{268}\rangle, H_{187}=\langle\sigma_{272}\rangle, H_{196}=\langle\sigma_{281}\rangle, H_{203}=\langle\sigma_{288}\rangle, H_{206}=\langle\sigma_{291}\rangle, H_{209}=\langle\sigma_{294}\rangle, H_{215}=\langle\sigma_{303}\rangle,$ $H_{221}=\langle\sigma_{310}\rangle, H_{225}=\langle\sigma_{316}\rangle, H_{232}=\langle\sigma_{325}\rangle, H_{241}=\langle\sigma_{336}\rangle, H_{249}=\langle\sigma_{350}\rangle, H_{257}=\langle\sigma_{359}\rangle, H_{265}=\langle\sigma_{398}\rangle, H_{267}=\langle\sigma_{402}\rangle,$ $H_{270}=\langle\sigma_{407}\rangle, H_{273}=\langle\sigma_{417}\rangle, H_{276}=\langle\sigma_{420}\rangle, H_{277}=\langle\sigma_{421}\rangle, H_{283}=\langle\sigma_{428}\rangle, H_{291}=\langle\sigma_{446}\rangle, H_{293}=\langle\sigma_{450}\rangle, H_{305}=\langle\sigma_{473}\rangle,$ $H_{316}=\langle\sigma_{520}\rangle, H_{317}=\langle\sigma_{521}\rangle, H_{321}=\langle\sigma_{538}\rangle, H_{324}=\langle\sigma_{543}\rangle, H_{330}=\langle\sigma_{551}\rangle, H_{334}=\langle\sigma_{568}\rangle, H_{335}=\langle\sigma_{569}\rangle, H_{349}=\langle\sigma_{642}\rangle,$ $H_{353}=\langle\sigma_{664}\rangle, H_{356}=\langle\sigma_{672}\rangle, H_{358}=\langle\sigma_{690}\rangle$
	$H_{94}=\langle\sigma_{154}\rangle, H_{95}=\langle\sigma_{155}\rangle, H_{98}=\langle\sigma_{158}\rangle, H_{102}=\langle\sigma_{162}\rangle, H_{103}=\langle\sigma_{163}\rangle, H_{107}=\langle\sigma_{167}\rangle, H_{112}=\langle\sigma_{172}\rangle, H_{113}=\langle\sigma_{173}\rangle,$ $H_{124}=\langle\sigma_{184}\rangle, H_{125}=\langle\sigma_{185}\rangle, H_{129}=\langle\sigma_{189}\rangle, H_{132}=\langle\sigma_{192}\rangle, H_{134}=\langle\sigma_{194}\rangle, H_{138}=\langle\sigma_{198}\rangle, H_{146}=\langle\sigma_{206}\rangle, H_{150}=\langle\sigma_{210}\rangle,$ $H_{151}=\langle\sigma_{211}\rangle, H_{154}=\langle\sigma_{214}\rangle, H_{157}=\langle\sigma_{217}\rangle, H_{161}=\langle\sigma_{221}\rangle, H_{169}=\langle\sigma_{229}\rangle, H_{173}=\langle\sigma_{233}\rangle, H_{176}=\langle\sigma_{236}\rangle, H_{177}=\langle\sigma_{237}\rangle,$ $H_{212}=\langle\sigma_{298}\rangle, H_{213}=\langle\sigma_{299}\rangle, H_{214}=\langle\sigma_{302}\rangle, H_{218}=\langle\sigma_{306}\rangle, H_{219}=\langle\sigma_{307}\rangle, H_{222}=\langle\sigma_{311}\rangle, H_{228}=\langle\sigma_{320}\rangle, H_{231}=\langle\sigma_{324}\rangle,$ $H_{234}=\langle\sigma_{328}\rangle, H_{235}=\langle\sigma_{329}\rangle, H_{238}=\langle\sigma_{332}\rangle, H_{239}=\langle\sigma_{333}\rangle, H_{244}=\langle\sigma_{343}\rangle, H_{246}=\langle\sigma_{347}\rangle, H_{250}=\langle\sigma_{351}\rangle, H_{252}=\langle\sigma_{354}\rangle,$ $H_{253}=\langle\sigma_{355}\rangle, H_{256}=\langle\sigma_{358}\rangle, H_{279}=\langle\sigma_{424}\rangle, H_{280}=\langle\sigma_{425}\rangle, H_{284}=\langle\sigma_{429}\rangle, H_{286}=\langle\sigma_{432}\rangle, H_{287}=\langle\sigma_{442}\rangle, H_{288}=\langle\sigma_{443}\rangle,$ $H_{297}=\langle\sigma_{454}\rangle, H_{298}=\langle\sigma_{455}\rangle, H_{300}=\langle\sigma_{465}\rangle, H_{302}=\langle\sigma_{468}\rangle, H_{307}=\langle\sigma_{477}\rangle, H_{310}=\langle\sigma_{480}\rangle, H_{327}=\langle\sigma_{546}\rangle, H_{329}=\langle\sigma_{550}\rangle,$ $H_{332}=\langle\sigma_{564}\rangle, H_{339}=\langle\sigma_{576}\rangle, H_{340}=\langle\sigma_{586}\rangle, H_{343}=\langle\sigma_{598}\rangle$
	$H_{363}=\langle\sigma_2, \sigma_3\rangle, H_{366}=\langle\sigma_2, \sigma_{15}\rangle, H_{375}=\langle\sigma_2, \sigma_{81}\rangle, H_{420}=\langle\sigma_2, \sigma_{513}\rangle, H_{431}=\langle\sigma_3, \sigma_7\rangle, H_{436}=\langle\sigma_3, \sigma_{55}\rangle,$ $H_{466}=\langle\sigma_3, \sigma_{391}\rangle, H_{503}=\langle\sigma_6, \sigma_7\rangle, H_{508}=\langle\sigma_6, \sigma_{55}\rangle, H_{538}=\langle\sigma_6, \sigma_{391}\rangle, H_{555}=\langle\sigma_7, \sigma_{25}\rangle, H_{569}=\langle\sigma_7, \sigma_{265}\rangle,$ $H_{683}=\langle\sigma_{15}, \sigma_{25}\rangle, H_{697}=\langle\sigma_{15}, \sigma_{265}\rangle, H_{772}=\langle\sigma_{22}, \sigma_{25}\rangle, H_{786}=\langle\sigma_{22}, \sigma_{265}\rangle, H_{841}=\langle\sigma_{25}, \sigma_{121}\rangle, H_{1001}=\langle\sigma_{55}, \sigma_{121}\rangle,$ $H_{1128}=\langle\sigma_{81}, \sigma_{121}\rangle, H_{1229}=\langle\sigma_{106}, \sigma_{121}\rangle$

位数	部分群
6	$H_{484}=\langle\sigma_4, \sigma_{26}\rangle, H_{487}=\langle\sigma_4, \sigma_{122}\rangle, H_{493}=\langle\sigma_4, \sigma_{266}\rangle, H_{591}=\langle\sigma_8, \sigma_{26}\rangle, H_{596}=\langle\sigma_8, \sigma_{82}\rangle, H_{612}=\langle\sigma_8, \sigma_{266}\rangle,$ $H_{629}=\langle\sigma_8, \sigma_{514}\rangle, H_{638}=\langle\sigma_9, \sigma_{108}\rangle, H_{639}=\langle\sigma_9, \sigma_{123}\rangle, H_{648}=\langle\sigma_9, \sigma_{636}\rangle, H_{670}=\langle\sigma_{12}, \sigma_{83}\rangle, H_{671}=\langle\sigma_{12}, \sigma_{126}\rangle,$ $H_{680}=\langle\sigma_{12}, \sigma_{515}\rangle, H_{719}=\langle\sigma_{16}, \sigma_{56}\rangle, H_{720}=\langle\sigma_{16}, \sigma_{122}\rangle, H_{729}=\langle\sigma_{16}, \sigma_{392}\rangle, H_{734}=\langle\sigma_{17}, \sigma_{30}\rangle, H_{735}=\langle\sigma_{17}, \sigma_{60}\rangle,$ $H_{750}=\langle\sigma_{17}, \sigma_{270}\rangle, H_{761}=\langle\sigma_{17}, \sigma_{396}\rangle, H_{808}=\langle\sigma_{24}, \sigma_{27}\rangle, H_{810}=\langle\sigma_{24}, \sigma_{57}\rangle, H_{823}=\langle\sigma_{24}, \sigma_{267}\rangle, H_{833}=\langle\sigma_{24}, \sigma_{393}\rangle,$ $H_{862}=\langle\sigma_{26}, \sigma_{122}\rangle, H_{885}=\langle\sigma_{26}, \sigma_{514}\rangle, H_{894}=\langle\sigma_{27}, \sigma_{123}\rangle, H_{910}=\langle\sigma_{27}, \sigma_{393}\rangle, H_{924}=\langle\sigma_{30}, \sigma_{126}\rangle, H_{940}=\langle\sigma_{30}, \sigma_{396}\rangle,$ $H_{955}=\langle\sigma_{31}, \sigma_{519}\rangle, H_{956}=\langle\sigma_{31}, \sigma_{640}\rangle, H_{977}=\langle\sigma_{39}, \sigma_{397}\rangle, H_{978}=\langle\sigma_{39}, \sigma_{648}\rangle, H_{999}=\langle\sigma_{46}, \sigma_{404}\rangle, H_{1000}=\langle\sigma_{46}, \sigma_{527}\rangle,$ $H_{1022}=\langle\sigma_{56}, \sigma_{122}\rangle, H_{1045}=\langle\sigma_{56}, \sigma_{514}\rangle, H_{1056}=\langle\sigma_{57}, \sigma_{267}\rangle, H_{1058}=\langle\sigma_{57}, \sigma_{636}\rangle, H_{1073}=\langle\sigma_{60}, \sigma_{270}\rangle, H_{1075}=\langle\sigma_{60}, \sigma_{515}\rangle,$ $H_{1081}=\langle\sigma_{61}, \sigma_{135}\rangle, H_{1090}=\langle\sigma_{61}, \sigma_{279}\rangle, H_{1109}=\langle\sigma_{68}, \sigma_{142}\rangle, H_{1117}=\langle\sigma_{68}, \sigma_{286}\rangle, H_{1152}=\langle\sigma_{82}, \sigma_{266}\rangle, H_{1154}=\langle\sigma_{82}, \sigma_{392}\rangle,$ $H_{1157}=\langle\sigma_{83}, \sigma_{126}\rangle, H_{1173}=\langle\sigma_{83}, \sigma_{396}\rangle, H_{1186}=\langle\sigma_{87}, \sigma_{127}\rangle, H_{1196}=\langle\sigma_{87}, \sigma_{271}\rangle, H_{1210}=\langle\sigma_{95}, \sigma_{142}\rangle, H_{1218}=\langle\sigma_{95}, \sigma_{286}\rangle,$ $H_{1250}=\langle\sigma_{108}, \sigma_{123}\rangle, H_{1264}=\langle\sigma_{108}, \sigma_{393}\rangle, H_{1274}=\langle\sigma_{112}, \sigma_{127}\rangle, H_{1283}=\langle\sigma_{112}, \sigma_{271}\rangle, H_{1296}=\langle\sigma_{120}, \sigma_{135}\rangle,$ $H_{1304}=\langle\sigma_{120}, \sigma_{279}\rangle$
	$H_{620}=\langle\sigma_8, \sigma_{315}\rangle, H_{625}=\langle\sigma_8, \sigma_{340}\rangle, H_{630}=\langle\sigma_8, \sigma_{537}\rangle, H_{633}=\langle\sigma_8, \sigma_{562}\rangle, H_{751}=\langle\sigma_{17}, \sigma_{289}\rangle, H_{759}=\langle\sigma_{17}, \sigma_{340}\rangle,$ $H_{762}=\langle\sigma_{17}, \sigma_{415}\rangle, H_{766}=\langle\sigma_{17}, \sigma_{444}\rangle, H_{825}=\langle\sigma_{24}, \sigma_{289}\rangle, H_{828}=\langle\sigma_{24}, \sigma_{315}\rangle, H_{834}=\langle\sigma_{24}, \sigma_{415}\rangle, H_{840}=\langle\sigma_{24}, \sigma_{467}\rangle,$ $H_{863}=\langle\sigma_{26}, \sigma_{123}\rangle, H_{865}=\langle\sigma_{26}, \sigma_{126}\rangle, H_{888}=\langle\sigma_{26}, \sigma_{519}\rangle, H_{889}=\langle\sigma_{26}, \sigma_{544}\rangle, H_{893}=\langle\sigma_{27}, \sigma_{122}\rangle, H_{895}=\langle\sigma_{27}, \sigma_{126}\rangle,$ $H_{912}=\langle\sigma_{27}, \sigma_{404}\rangle, H_{913}=\langle\sigma_{27}, \sigma_{431}\rangle, H_{922}=\langle\sigma_{30}, \sigma_{122}\rangle, H_{923}=\langle\sigma_{30}, \sigma_{123}\rangle, H_{941}=\langle\sigma_{30}, \sigma_{397}\rangle, H_{942}=\langle\sigma_{30}, \sigma_{426}\rangle,$ $H_{1027}=\langle\sigma_{56}, \sigma_{135}\rangle, H_{1030}=\langle\sigma_{56}, \sigma_{142}\rangle, H_{1046}=\langle\sigma_{56}, \sigma_{519}\rangle, H_{1049}=\langle\sigma_{56}, \sigma_{537}\rangle, H_{1076}=\langle\sigma_{61}, \sigma_{122}\rangle, H_{1083}=\langle\sigma_{61}, \sigma_{142}\rangle,$ $H_{1089}=\langle\sigma_{61}, \sigma_{270}\rangle, H_{1092}=\langle\sigma_{61}, \sigma_{305}\rangle, H_{1102}=\langle\sigma_{68}, \sigma_{122}\rangle, H_{1107}=\langle\sigma_{68}, \sigma_{135}\rangle, H_{1115}=\langle\sigma_{68}, \sigma_{267}\rangle, H_{1118}=\langle\sigma_{68}, \sigma_{312}\rangle,$ $H_{1158}=\langle\sigma_{83}, \sigma_{127}\rangle, H_{1163}=\langle\sigma_{83}, \sigma_{142}\rangle, H_{1174}=\langle\sigma_{83}, \sigma_{397}\rangle, H_{1177}=\langle\sigma_{83}, \sigma_{415}\rangle, H_{1185}=\langle\sigma_{87}, \sigma_{126}\rangle, H_{1190}=\langle\sigma_{87}, \sigma_{142}\rangle,$ $H_{1193}=\langle\sigma_{87}, \sigma_{266}\rangle, H_{1199}=\langle\sigma_{87}, \sigma_{326}\rangle, H_{1205}=\langle\sigma_{95}, \sigma_{126}\rangle, H_{1206}=\langle\sigma_{95}, \sigma_{127}\rangle, H_{1215}=\langle\sigma_{95}, \sigma_{267}\rangle, H_{1219}=\langle\sigma_{95}, \sigma_{289}\rangle,$ $H_{1252}=\langle\sigma_{108}, \sigma_{127}\rangle, H_{1255}=\langle\sigma_{108}, \sigma_{135}\rangle, H_{1267}=\langle\sigma_{108}, \sigma_{404}\rangle, H_{1268}=\langle\sigma_{108}, \sigma_{415}\rangle, H_{1272}=\langle\sigma_{112}, \sigma_{123}\rangle,$ $H_{1276}=\langle\sigma_{112}, \sigma_{135}\rangle, H_{1281}=\langle\sigma_{112}, \sigma_{266}\rangle, H_{1287}=\langle\sigma_{112}, \sigma_{315}\rangle, H_{1292}=\langle\sigma_{120}, \sigma_{123}\rangle, H_{1294}=\langle\sigma_{120}, \sigma_{127}\rangle,$ $H_{1302}=\langle\sigma_{120}, \sigma_{270}\rangle, H_{1305}=\langle\sigma_{120}, \sigma_{289}\rangle$
	$H_{1321}=\langle\sigma_{128}, \sigma_{317}\rangle, H_{1322}=\langle\sigma_{128}, \sigma_{334}\rangle, H_{1323}=\langle\sigma_{128}, \sigma_{342}\rangle, H_{1324}=\langle\sigma_{128}, \sigma_{357}\rangle, H_{1329}=\langle\sigma_{137}, \sigma_{290}\rangle,$ $H_{1330}=\langle\sigma_{137}, \sigma_{312}\rangle, H_{1333}=\langle\sigma_{137}, \sigma_{342}\rangle, H_{1334}=\langle\sigma_{137}, \sigma_{349}\rangle, H_{1339}=\langle\sigma_{144}, \sigma_{290}\rangle, H_{1340}=\langle\sigma_{144}, \sigma_{305}\rangle,$ $H_{1341}=\langle\sigma_{144}, \sigma_{317}\rangle, H_{1342}=\langle\sigma_{144}, \sigma_{326}\rangle, H_{1347}=\langle\sigma_{148}, \sigma_{476}\rangle, H_{1348}=\langle\sigma_{149}, \sigma_{451}\rangle, H_{1349}=\langle\sigma_{182}, \sigma_{342}\rangle,$ $H_{1350}=\langle\sigma_{187}, \sigma_{317}\rangle, H_{1351}=\langle\sigma_{209}, \sigma_{342}\rangle, H_{1352}=\langle\sigma_{213}, \sigma_{290}\rangle, H_{1353}=\langle\sigma_{234}, \sigma_{317}\rangle, H_{1354}=\langle\sigma_{238}, \sigma_{290}\rangle$
8	$H_{367}=\langle\sigma_2, \sigma_{17}\rangle, H_{376}=\langle\sigma_2, \sigma_{83}\rangle, H_{378}=\langle\sigma_2, \sigma_{95}\rangle, H_{421}=\langle\sigma_2, \sigma_{515}\rangle, H_{423}=\langle\sigma_2, \sigma_{527}\rangle, H_{429}=\langle\sigma_2, \sigma_{593}\rangle,$ $H_{432}=\langle\sigma_3, \sigma_8\rangle, H_{437}=\langle\sigma_3, \sigma_{56}\rangle, H_{438}=\langle\sigma_3, \sigma_{61}\rangle, H_{467}=\langle\sigma_3, \sigma_{392}\rangle, H_{468}=\langle\sigma_3, \sigma_{397}\rangle, H_{473}=\langle\sigma_3, \sigma_{445}\rangle,$ $H_{504}=\langle\sigma_6, \sigma_8\rangle, H_{509}=\langle\sigma_6, \sigma_{56}\rangle, H_{511}=\langle\sigma_6, \sigma_{68}\rangle, H_{539}=\langle\sigma_6, \sigma_{392}\rangle, H_{541}=\langle\sigma_6, \sigma_{404}\rangle, H_{547}=\langle\sigma_6, \sigma_{470}\rangle,$ $H_{557}=\langle\sigma_7, \sigma_{27}\rangle, H_{558}=\langle\sigma_7, \sigma_{30}\rangle, H_{571}=\langle\sigma_7, \sigma_{267}\rangle, H_{572}=\langle\sigma_7, \sigma_{270}\rangle, H_{573}=\langle\sigma_7, \sigma_{289}\rangle, H_{684}=\langle\sigma_{15}, \sigma_{26}\rangle,$ $H_{685}=\langle\sigma_{15}, \sigma_{27}\rangle, H_{698}=\langle\sigma_{15}, \sigma_{266}\rangle, H_{699}=\langle\sigma_{15}, \sigma_{267}\rangle, H_{704}=\langle\sigma_{15}, \sigma_{315}\rangle, H_{773}=\langle\sigma_{22}, \sigma_{26}\rangle, H_{775}=\langle\sigma_{22}, \sigma_{30}\rangle,$ $H_{787}=\langle\sigma_{22}, \sigma_{266}\rangle, H_{789}=\langle\sigma_{22}, \sigma_{270}\rangle, H_{795}=\langle\sigma_{22}, \sigma_{340}\rangle, H_{846}=\langle\sigma_{25}, \sigma_{127}\rangle, H_{848}=\langle\sigma_{25}, \sigma_{135}\rangle, H_{850}=\langle\sigma_{25}, \sigma_{142}\rangle,$ $H_{1003}=\langle\sigma_{55}, \sigma_{123}\rangle, H_{1004}=\langle\sigma_{55}, \sigma_{126}\rangle, H_{1005}=\langle\sigma_{55}, \sigma_{127}\rangle, H_{1129}=\langle\sigma_{81}, \sigma_{122}\rangle, H_{1130}=\langle\sigma_{81}, \sigma_{123}\rangle, H_{1135}=\langle\sigma_{81}, \sigma_{135}\rangle$ $H_{1230}=\langle\sigma_{106}, \sigma_{122}\rangle, H_{1232}=\langle\sigma_{106}, \sigma_{126}\rangle, H_{1238}=\langle\sigma_{106}, \sigma_{142}\rangle$
	$H_{384}=\langle\sigma_2, \sigma_{137}\rangle, H_{408}=\langle\sigma_2, \sigma_{317}\rangle, H_{418}=\langle\sigma_2, \sigma_{451}\rangle, H_{425}=\langle\sigma_2, \sigma_{539}\rangle, H_{428}=\langle\sigma_2, \sigma_{573}\rangle, H_{430}=\langle\sigma_2, \sigma_{599}\rangle,$ $H_{444}=\langle\sigma_3, \sigma_{128}\rangle, H_{459}=\langle\sigma_3, \sigma_{290}\rangle, H_{471}=\langle\sigma_3, \sigma_{416}\rangle, H_{474}=\langle\sigma_3, \sigma_{451}\rangle, H_{476}=\langle\sigma_3, \sigma_{476}\rangle, H_{482}=\langle\sigma_3, \sigma_{694}\rangle,$ $H_{516}=\langle\sigma_6, \sigma_{128}\rangle, H_{531}=\langle\sigma_6, \sigma_{290}\rangle, H_{543}=\langle\sigma_6, \sigma_{416}\rangle, H_{546}=\langle\sigma_6, \sigma_{451}\rangle, H_{548}=\langle\sigma_6, \sigma_{476}\rangle, H_{554}=\langle\sigma_6, \sigma_{573}\rangle,$ $H_{574}=\langle\sigma_7, \sigma_{290}\rangle, H_{577}=\langle\sigma_7, \sigma_{317}\rangle, H_{579}=\langle\sigma_7, \sigma_{342}\rangle, H_{583}=\langle\sigma_7, \sigma_{539}\rangle, H_{588}=\langle\sigma_7, \sigma_{660}\rangle, H_{702}=\langle\sigma_{15}, \sigma_{290}\rangle,$ $H_{705}=\langle\sigma_{15}, \sigma_{317}\rangle, H_{707}=\langle\sigma_{15}, \sigma_{342}\rangle, H_{710}=\langle\sigma_{15}, \sigma_{416}\rangle, H_{716}=\langle\sigma_{15}, \sigma_{660}\rangle, H_{791}=\langle\sigma_{22}, \sigma_{290}\rangle, H_{794}=\langle\sigma_{22}, \sigma_{317}\rangle,$ $H_{796}=\langle\sigma_{22}, \sigma_{342}\rangle, H_{799}=\langle\sigma_{22}, \sigma_{416}\rangle, H_{805}=\langle\sigma_{22}, \sigma_{539}\rangle, H_{847}=\langle\sigma_{25}, \sigma_{128}\rangle, H_{849}=\langle\sigma_{25}, \sigma_{137}\rangle, H_{851}=\langle\sigma_{25}, \sigma_{144}\rangle,$ $H_{1006}=\langle\sigma_{55}, \sigma_{128}\rangle, H_{1009}=\langle\sigma_{55}, \sigma_{137}\rangle, H_{1011}=\langle\sigma_{55}, \sigma_{144}\rangle, H_{1133}=\langle\sigma_{81}, \sigma_{128}\rangle, H_{1136}=\langle\sigma_{81}, \sigma_{137}\rangle, H_{1138}=\langle\sigma_{81}, \sigma_{144}\rangle,$ $H_{1234}=\langle\sigma_{106}, \sigma_{128}\rangle, H_{1237}=\langle\sigma_{106}, \sigma_{137}\rangle, H_{1239}=\langle\sigma_{106}, \sigma_{144}\rangle$

位数	部分群
8	$H_{386}=\langle\sigma_2, \sigma_{151}\rangle, H_{390}=\langle\sigma_2, \sigma_{169}\rangle, H_{404}=\langle\sigma_2, \sigma_{295}\rangle, H_{448}=\langle\sigma_3, \sigma_{166}\rangle, H_{454}=\langle\sigma_3, \sigma_{220}\rangle, H_{463}=\langle\sigma_3, \sigma_{346}\rangle,$ $H_{520}=\langle\sigma_6, \sigma_{159}\rangle, H_{526}=\langle\sigma_6, \sigma_{195}\rangle, H_{535}=\langle\sigma_6, \sigma_{321}\rangle, H_{567}=\langle\sigma_7, \sigma_{202}\rangle, H_{568}=\langle\sigma_7, \sigma_{225}\rangle, H_{585}=\langle\sigma_7, \sigma_{594}\rangle,$ $H_{653}=\langle\sigma_{10}, \sigma_{121}\rangle, H_{662}=\langle\sigma_{11}, \sigma_{121}\rangle, H_{695}=\langle\sigma_{15}, \sigma_{180}\rangle, H_{696}=\langle\sigma_{15}, \sigma_{224}\rangle, H_{713}=\langle\sigma_{15}, \sigma_{472}\rangle, H_{769}=\langle\sigma_{18}, \sigma_{121}\rangle,$ $H_{784}=\langle\sigma_{22}, \sigma_{177}\rangle, H_{785}=\langle\sigma_{22}, \sigma_{199}\rangle, H_{802}=\langle\sigma_{22}, \sigma_{447}\rangle, H_{854}=\langle\sigma_{25}, \sigma_{394}\rangle, H_{855}=\langle\sigma_{25}, \sigma_{395}\rangle, H_{858}=\langle\sigma_{25}, \sigma_{516}\rangle,$ $H_{960}=\langle\sigma_{33}, \sigma_{634}\rangle, H_{965}=\langle\sigma_{36}, \sigma_{513}\rangle, H_{970}=\langle\sigma_{37}, \sigma_{634}\rangle, H_{982}=\langle\sigma_{40}, \sigma_{391}\rangle, H_{987}=\langle\sigma_{44}, \sigma_{513}\rangle, H_{992}=\langle\sigma_{45}, \sigma_{391}\rangle,$ $H_{1014}=\langle\sigma_{55}, \sigma_{280}\rangle, H_{1015}=\langle\sigma_{55}, \sigma_{285}\rangle, H_{1018}=\langle\sigma_{55}, \sigma_{528}\rangle, H_{1062}=\langle\sigma_{58}, \sigma_{265}\rangle, H_{1067}=\langle\sigma_{59}, \sigma_{265}\rangle, H_{1100}=\langle\sigma_{63}, \sigma_{634}\rangle,$ $H_{1126}=\langle\sigma_{70}, \sigma_{513}\rangle, H_{1141}=\langle\sigma_{81}, \sigma_{276}\rangle, H_{1142}=\langle\sigma_{81}, \sigma_{284}\rangle, H_{1145}=\langle\sigma_{81}, \sigma_{406}\rangle, H_{1181}=\langle\sigma_{84}, \sigma_{265}\rangle, H_{1227}=\langle\sigma_{96}, \sigma_{391}\rangle,$ $H_{1242}=\langle\sigma_{106}, \sigma_{273}\rangle, H_{1243}=\langle\sigma_{106}, \sigma_{277}\rangle, H_{1246}=\langle\sigma_{106}, \sigma_{399}\rangle$
	$H_{599}=\langle\sigma_8, \sigma_{123}\rangle, H_{600}=\langle\sigma_8, \sigma_{126}\rangle, H_{617}=\langle\sigma_8, \sigma_{289}\rangle, H_{635}=\langle\sigma_8, \sigma_{593}\rangle, H_{737}=\langle\sigma_{17}, \sigma_{122}\rangle, H_{756}=\langle\sigma_{17}, \sigma_{315}\rangle,$ $H_{767}=\langle\sigma_{17}, \sigma_{470}\rangle, H_{830}=\langle\sigma_{24}, \sigma_{340}\rangle, H_{837}=\langle\sigma_{24}, \sigma_{445}\rangle, H_{866}=\langle\sigma_{26}, \sigma_{127}\rangle, H_{881}=\langle\sigma_{26}, \sigma_{397}\rangle, H_{882}=\langle\sigma_{26}, \sigma_{404}\rangle,$ $H_{886}=\langle\sigma_{26}, \sigma_{515}\rangle, H_{900}=\langle\sigma_{27}, \sigma_{142}\rangle, H_{909}=\langle\sigma_{27}, \sigma_{392}\rangle, H_{918}=\langle\sigma_{27}, \sigma_{640}\rangle, H_{919}=\langle\sigma_{27}, \sigma_{648}\rangle, H_{927}=\langle\sigma_{30}, \sigma_{135}\rangle,$ $H_{938}=\langle\sigma_{30}, \sigma_{392}\rangle, H_{947}=\langle\sigma_{30}, \sigma_{519}\rangle, H_{948}=\langle\sigma_{30}, \sigma_{527}\rangle, H_{1025}=\langle\sigma_{56}, \sigma_{127}\rangle, H_{1041}=\langle\sigma_{56}, \sigma_{267}\rangle, H_{1042}=\langle\sigma_{56}, \sigma_{270}\rangle,$ $H_{1047}=\langle\sigma_{56}, \sigma_{527}\rangle, H_{1078}=\langle\sigma_{61}, \sigma_{126}\rangle, H_{1088}=\langle\sigma_{61}, \sigma_{266}\rangle, H_{1096}=\langle\sigma_{61}, \sigma_{636}\rangle, H_{1103}=\langle\sigma_{68}, \sigma_{123}\rangle, H_{1114}=\langle\sigma_{68}, \sigma_{266}\rangle,$ $H_{1122}=\langle\sigma_{68}, \sigma_{515}\rangle, H_{1161}=\langle\sigma_{83}, \sigma_{135}\rangle, H_{1169}=\langle\sigma_{83}, \sigma_{266}\rangle, H_{1175}=\langle\sigma_{83}, \sigma_{404}\rangle, H_{1183}=\langle\sigma_{87}, \sigma_{122}\rangle, H_{1194}=\langle\sigma_{87}, \sigma_{270}\rangle,$ $H_{1204}=\langle\sigma_{95}, \sigma_{123}\rangle, H_{1216}=\langle\sigma_{95}, \sigma_{270}\rangle, H_{1223}=\langle\sigma_{95}, \sigma_{392}\rangle, H_{1257}=\langle\sigma_{108}, \sigma_{142}\rangle, H_{1265}=\langle\sigma_{108}, \sigma_{397}\rangle,$ $H_{1271}=\langle\sigma_{112}, \sigma_{122}\rangle, H_{1282}=\langle\sigma_{112}, \sigma_{267}\rangle, H_{1293}=\langle\sigma_{120}, \sigma_{126}\rangle, H_{1301}=\langle\sigma_{120}, \sigma_{267}\rangle$
	$H_{618}=\langle\sigma_8, \sigma_{290}\rangle, H_{636}=\langle\sigma_8, \sigma_{594}\rangle, H_{654}=\langle\sigma_{10}, \sigma_{126}\rangle, H_{663}=\langle\sigma_{11}, \sigma_{123}\rangle, H_{757}=\langle\sigma_{17}, \sigma_{317}\rangle, H_{768}=\langle\sigma_{17}, \sigma_{472}\rangle,$ $H_{770}=\langle\sigma_{18}, \sigma_{122}\rangle, H_{831}=\langle\sigma_{24}, \sigma_{342}\rangle, H_{838}=\langle\sigma_{24}, \sigma_{447}\rangle, H_{867}=\langle\sigma_{26}, \sigma_{128}\rangle, H_{887}=\langle\sigma_{26}, \sigma_{516}\rangle, H_{901}=\langle\sigma_{27}, \sigma_{144}\rangle,$ $H_{911}=\langle\sigma_{27}, \sigma_{395}\rangle, H_{928}=\langle\sigma_{30}, \sigma_{137}\rangle, H_{939}=\langle\sigma_{30}, \sigma_{394}\rangle, H_{961}=\langle\sigma_{33}, \sigma_{648}\rangle, H_{966}=\langle\sigma_{36}, \sigma_{527}\rangle, H_{971}=\langle\sigma_{37}, \sigma_{640}\rangle,$ $H_{983}=\langle\sigma_{40}, \sigma_{404}\rangle, H_{988}=\langle\sigma_{44}, \sigma_{519}\rangle, H_{993}=\langle\sigma_{45}, \sigma_{397}\rangle, H_{1026}=\langle\sigma_{56}, \sigma_{128}\rangle, H_{1048}=\langle\sigma_{56}, \sigma_{528}\rangle, H_{1063}=\langle\sigma_{58}, \sigma_{270}\rangle,$ $H_{1068}=\langle\sigma_{59}, \sigma_{267}\rangle, H_{1082}=\langle\sigma_{61}, \sigma_{137}\rangle, H_{1091}=\langle\sigma_{61}, \sigma_{285}\rangle, H_{1101}=\langle\sigma_{63}, \sigma_{636}\rangle, H_{1110}=\langle\sigma_{68}, \sigma_{144}\rangle, H_{1116}=\langle\sigma_{68}, \sigma_{280}\rangle,$ $H_{1127}=\langle\sigma_{70}, \sigma_{515}\rangle, H_{1162}=\langle\sigma_{83}, \sigma_{137}\rangle, H_{1176}=\langle\sigma_{83}, \sigma_{406}\rangle, H_{1182}=\langle\sigma_{84}, \sigma_{266}\rangle, H_{1187}=\langle\sigma_{87}, \sigma_{128}\rangle, H_{1196}=\langle\sigma_{87}, \sigma_{284}\rangle,$ $H_{1211}=\langle\sigma_{95}, \sigma_{144}\rangle, H_{1217}=\langle\sigma_{95}, \sigma_{276}\rangle, H_{1228}=\langle\sigma_{96}, \sigma_{392}\rangle, H_{1258}=\langle\sigma_{108}, \sigma_{144}\rangle, H_{1266}=\langle\sigma_{108}, \sigma_{399}\rangle,$ $H_{1275}=\langle\sigma_{112}, \sigma_{128}\rangle, H_{1284}=\langle\sigma_{112}, \sigma_{277}\rangle, H_{1297}=\langle\sigma_{120}, \sigma_{137}\rangle, H_{1303}=\langle\sigma_{120}, \sigma_{273}\rangle$
	$H_{1371}=\langle\sigma_2, \sigma_7, \sigma_{121}\rangle, H_{1376}=\langle\sigma_2, \sigma_{25}, \sigma_{391}\rangle, H_{1379}=\langle\sigma_2, \sigma_{55}, \sigma_{265}\rangle, H_{1385}=\langle\sigma_3, \sigma_{22}, \sigma_{121}\rangle, H_{1390}=\langle\sigma_3, \sigma_{25}, \sigma_{634}\rangle,$ $H_{1394}=\langle\sigma_3, \sigma_{106}, \sigma_{265}\rangle, H_{1399}=\langle\sigma_6, \sigma_{15}, \sigma_{121}\rangle, H_{1404}=\langle\sigma_6, \sigma_{25}, \sigma_{513}\rangle, H_{1408}=\langle\sigma_6, \sigma_{81}, \sigma_{265}\rangle, H_{1415}=\langle\sigma_7, \sigma_{81}, \sigma_{634}\rangle,$ $H_{1418}=\langle\sigma_7, \sigma_{106}, \sigma_{513}\rangle, H_{1429}=\langle\sigma_{15}, \sigma_{55}, \sigma_{634}\rangle, H_{1432}=\langle\sigma_{15}, \sigma_{106}, \sigma_{391}\rangle, H_{1441}=\langle\sigma_{22}, \sigma_{55}, \sigma_{513}\rangle,$ $H_{1444}=\langle\sigma_{22}, \sigma_{81}, \sigma_{391}\rangle$
	$H_{1383}=\langle\sigma_2, \sigma_{127}, \sigma_{289}\rangle, H_{1396}=\langle\sigma_3, \sigma_{142}, \sigma_{340}\rangle, H_{1410}=\langle\sigma_6, \sigma_{135}, \sigma_{315}\rangle, H_{1419}=\langle\sigma_7, \sigma_{122}, \sigma_{593}\rangle,$ $H_{1420}=\langle\sigma_8, \sigma_{17}, \sigma_{121}\rangle, H_{1433}=\langle\sigma_{15}, \sigma_{126}, \sigma_{470}\rangle, H_{1445}=\langle\sigma_{22}, \sigma_{123}, \sigma_{445}\rangle, H_{1448}=\langle\sigma_{25}, \sigma_{392}, \sigma_{515}\rangle,$ $H_{1449}=\langle\sigma_{26}, \sigma_{95}, \sigma_{391}\rangle, H_{1450}=\langle\sigma_{27}, \sigma_{61}, \sigma_{634}\rangle, H_{1451}=\langle\sigma_{30}, \sigma_{68}, \sigma_{513}\rangle, H_{1452}=\langle\sigma_{55}, \sigma_{266}, \sigma_{527}\rangle,$ $H_{1453}=\langle\sigma_{56}, \sigma_{83}, \sigma_{265}\rangle, H_{1454}=\langle\sigma_{81}, \sigma_{270}, \sigma_{404}\rangle, H_{1455}=\langle\sigma_{106}, \sigma_{267}, \sigma_{397}\rangle$
9	$H_{490}=\langle\sigma_4, \sigma_{145}\rangle, H_{641}=\langle\sigma_9, \sigma_{226}\rangle, H_{673}=\langle\sigma_{12}, \sigma_{201}\rangle, H_{722}=\langle\sigma_{16}, \sigma_{175}\rangle, H_{954}=\langle\sigma_{31}, \sigma_{514}\rangle, H_{976}=\langle\sigma_{39}, \sigma_{396}\rangle,$ $H_{998}=\langle\sigma_{46}, \sigma_{393}\rangle, H_{1057}=\langle\sigma_{57}, \sigma_{286}\rangle, H_{1074}=\langle\sigma_{60}, \sigma_{279}\rangle, H_{1153}=\langle\sigma_{82}, \sigma_{271}\rangle$
10	$H_{592}=\langle\sigma_8, \sigma_{27}\rangle, H_{593}=\langle\sigma_8, \sigma_{30}\rangle, H_{613}=\langle\sigma_8, \sigma_{267}\rangle, H_{614}=\langle\sigma_8, \sigma_{270}\rangle, H_{732}=\langle\sigma_{17}, \sigma_{26}\rangle, H_{733}=\langle\sigma_{17}, \sigma_{27}\rangle,$ $H_{748}=\langle\sigma_{17}, \sigma_{266}\rangle, H_{749}=\langle\sigma_{17}, \sigma_{267}\rangle, H_{807}=\langle\sigma_{24}, \sigma_{26}\rangle, H_{809}=\langle\sigma_{24}, \sigma_{30}\rangle, H_{822}=\langle\sigma_{24}, \sigma_{266}\rangle, H_{824}=\langle\sigma_{24}, \sigma_{270}\rangle,$ $H_{868}=\langle\sigma_{26}, \sigma_{135}\rangle, H_{870}=\langle\sigma_{26}, \sigma_{142}\rangle, H_{896}=\langle\sigma_{27}, \sigma_{127}\rangle, H_{898}=\langle\sigma_{27}, \sigma_{135}\rangle, H_{925}=\langle\sigma_{30}, \sigma_{127}\rangle, H_{929}=\langle\sigma_{30}, \sigma_{142}\rangle,$ $H_{1023}=\langle\sigma_{56}, \sigma_{123}\rangle, H_{1024}=\langle\sigma_{56}, \sigma_{126}\rangle, H_{1077}=\langle\sigma_{61}, \sigma_{123}\rangle, H_{1079}=\langle\sigma_{61}, \sigma_{127}\rangle, H_{1104}=\langle\sigma_{68}, \sigma_{126}\rangle, H_{1105}=\langle\sigma_{68}, \sigma_{127}\rangle,$ $H_{1155}=\langle\sigma_{83}, \sigma_{122}\rangle, H_{1156}=\langle\sigma_{83}, \sigma_{123}\rangle, H_{1184}=\langle\sigma_{87}, \sigma_{123}\rangle, H_{1188}=\langle\sigma_{87}, \sigma_{135}\rangle, H_{1203}=\langle\sigma_{95}, \sigma_{122}\rangle, H_{1208}=\langle\sigma_{95}, \sigma_{135}\rangle,$ $H_{1249}=\langle\sigma_{108}, \sigma_{122}\rangle, H_{1251}=\langle\sigma_{108}, \sigma_{126}\rangle, H_{1273}=\langle\sigma_{112}, \sigma_{126}\rangle, H_{1278}=\langle\sigma_{112}, \sigma_{142}\rangle, H_{1291}=\langle\sigma_{120}, \sigma_{122}\rangle,$ $H_{1298}=\langle\sigma_{120}, \sigma_{142}\rangle$

位数	部分群
12	$H_{369}=\langle\sigma_2, \sigma_{27}\rangle, H_{374}=\langle\sigma_2, \sigma_{61}\rangle, H_{377}=\langle\sigma_2, \sigma_{87}\rangle, H_{380}=\langle\sigma_2, \sigma_{123}\rangle, H_{383}=\langle\sigma_2, \sigma_{135}\rangle, H_{399}=\langle\sigma_2, \sigma_{267}\rangle,$ $H_{407}=\langle\sigma_2, \sigma_{315}\rangle, H_{413}=\langle\sigma_2, \sigma_{397}\rangle, H_{417}=\langle\sigma_2, \sigma_{445}\rangle, H_{422}=\langle\sigma_2, \sigma_{519}\rangle, H_{424}=\langle\sigma_2, \sigma_{537}\rangle, H_{427}=\langle\sigma_2, \sigma_{567}\rangle,$ $H_{439}=\langle\sigma_3, \sigma_{68}\rangle, H_{441}=\langle\sigma_3, \sigma_{112}\rangle, H_{443}=\langle\sigma_3, \sigma_{127}\rangle, H_{458}=\langle\sigma_3, \sigma_{289}\rangle, H_{469}=\langle\sigma_3, \sigma_{404}\rangle, H_{470}=\langle\sigma_3, \sigma_{415}\rangle,$ $H_{475}=\langle\sigma_3, \sigma_{470}\rangle, H_{478}=\langle\sigma_3, \sigma_{640}\rangle, H_{481}=\langle\sigma_3, \sigma_{688}\rangle, H_{510}=\langle\sigma_6, \sigma_{61}\rangle, H_{513}=\langle\sigma_6, \sigma_{87}\rangle, H_{515}=\langle\sigma_6, \sigma_{127}\rangle,$ $H_{530}=\langle\sigma_6, \sigma_{289}\rangle, H_{540}=\langle\sigma_6, \sigma_{397}\rangle, H_{542}=\langle\sigma_6, \sigma_{415}\rangle, H_{545}=\langle\sigma_6, \sigma_{445}\rangle, H_{550}=\langle\sigma_6, \sigma_{519}\rangle, H_{553}=\langle\sigma_6, \sigma_{567}\rangle,$ $H_{556}=\langle\sigma_7, \sigma_{26}\rangle, H_{570}=\langle\sigma_7, \sigma_{266}\rangle, H_{575}=\langle\sigma_7, \sigma_{315}\rangle, H_{578}=\langle\sigma_7, \sigma_{340}\rangle, H_{581}=\langle\sigma_7, \sigma_{537}\rangle, H_{587}=\langle\sigma_7, \sigma_{658}\rangle,$ $H_{686}=\langle\sigma_{15}, \sigma_{30}\rangle, H_{700}=\langle\sigma_{15}, \sigma_{270}\rangle, H_{701}=\langle\sigma_{15}, \sigma_{289}\rangle, H_{706}=\langle\sigma_{15}, \sigma_{340}\rangle, H_{709}=\langle\sigma_{15}, \sigma_{415}\rangle, H_{715}=\langle\sigma_{15}, \sigma_{658}\rangle,$ $H_{774}=\langle\sigma_{22}, \sigma_{27}\rangle, H_{788}=\langle\sigma_{22}, \sigma_{267}\rangle, H_{790}=\langle\sigma_{22}, \sigma_{289}\rangle, H_{793}=\langle\sigma_{22}, \sigma_{315}\rangle, H_{798}=\langle\sigma_{22}, \sigma_{415}\rangle, H_{804}=\langle\sigma_{22}, \sigma_{537}\rangle,$ $H_{842}=\langle\sigma_{25}, \sigma_{122}\rangle, H_{843}=\langle\sigma_{25}, \sigma_{123}\rangle, H_{845}=\langle\sigma_{25}, \sigma_{126}\rangle, H_{1002}=\langle\sigma_{55}, \sigma_{122}\rangle, H_{1007}=\langle\sigma_{55}, \sigma_{135}\rangle, H_{1010}=\langle\sigma_{55}, \sigma_{142}\rangle,$ $H_{1131}=\langle\sigma_{81}, \sigma_{126}\rangle, H_{1132}=\langle\sigma_{81}, \sigma_{127}\rangle, H_{1137}=\langle\sigma_{81}, \sigma_{142}\rangle, H_{1231}=\langle\sigma_{106}, \sigma_{123}\rangle, H_{1233}=\langle\sigma_{106}, \sigma_{127}\rangle,$ $H_{1236}=\langle\sigma_{106}, \sigma_{135}\rangle$
	$H_{483}=\langle\sigma_4, \sigma_8\rangle, H_{486}=\langle\sigma_4, \sigma_{56}\rangle, H_{497}=\langle\sigma_4, \sigma_{392}\rangle, H_{637}=\langle\sigma_9, \sigma_{27}\rangle, H_{642}=\langle\sigma_9, \sigma_{267}\rangle, H_{669}=\langle\sigma_{12}, \sigma_{30}\rangle,$ $H_{674}=\langle\sigma_{12}, \sigma_{270}\rangle, H_{718}=\langle\sigma_{16}, \sigma_{26}\rangle, H_{723}=\langle\sigma_{16}, \sigma_{266}\rangle, H_{950}=\langle\sigma_{31}, \sigma_{127}\rangle, H_{973}=\langle\sigma_{39}, \sigma_{135}\rangle, H_{996}=\langle\sigma_{46}, \sigma_{142}\rangle,$ $H_{1052}=\langle\sigma_{57}, \sigma_{123}\rangle, H_{1069}=\langle\sigma_{60}, \sigma_{126}\rangle, H_{1148}=\langle\sigma_{82}, \sigma_{122}\rangle$
	$H_{619}=\langle\sigma_8, \sigma_{305}\rangle, H_{758}=\langle\sigma_{17}, \sigma_{326}\rangle, H_{832}=\langle\sigma_{24}, \sigma_{349}\rangle, H_{878}=\langle\sigma_{26}, \sigma_{213}\rangle, H_{906}=\langle\sigma_{27}, \sigma_{182}\rangle, H_{936}=\langle\sigma_{30}, \sigma_{187}\rangle,$ $H_{1038}=\langle\sigma_{56}, \sigma_{209}\rangle, H_{1085}=\langle\sigma_{61}, \sigma_{148}\rangle, H_{1111}=\langle\sigma_{68}, \sigma_{149}\rangle, H_{1167}=\langle\sigma_{83}, \sigma_{182}\rangle, H_{1192}=\langle\sigma_{87}, \sigma_{149}\rangle, H_{1212}=\langle\sigma_{95}, \sigma_{148}\rangle,$ $H_{1259}=\langle\sigma_{108}, \sigma_{187}\rangle, H_{1280}=\langle\sigma_{112}, \sigma_{148}\rangle, H_{1300}=\langle\sigma_{120}, \sigma_{149}\rangle$
	$H_{622}=\langle\sigma_8, \sigma_{317}\rangle, H_{626}=\langle\sigma_8, \sigma_{342}\rangle, H_{632}=\langle\sigma_8, \sigma_{539}\rangle, H_{634}=\langle\sigma_8, \sigma_{564}\rangle, H_{752}=\langle\sigma_{17}, \sigma_{290}\rangle, H_{760}=\langle\sigma_{17}, \sigma_{342}\rangle,$ $H_{763}=\langle\sigma_{17}, \sigma_{416}\rangle, H_{765}=\langle\sigma_{17}, \sigma_{442}\rangle, H_{826}=\langle\sigma_{24}, \sigma_{290}\rangle, H_{829}=\langle\sigma_{24}, \sigma_{317}\rangle, H_{835}=\langle\sigma_{24}, \sigma_{416}\rangle, H_{839}=\langle\sigma_{24}, \sigma_{465}\rangle,$ $H_{869}=\langle\sigma_{26}, \sigma_{137}\rangle, H_{871}=\langle\sigma_{26}, \sigma_{144}\rangle, H_{890}=\langle\sigma_{26}, \sigma_{573}\rangle, H_{891}=\langle\sigma_{26}, \sigma_{576}\rangle, H_{897}=\langle\sigma_{27}, \sigma_{128}\rangle, H_{899}=\langle\sigma_{27}, \sigma_{137}\rangle,$ $H_{914}=\langle\sigma_{27}, \sigma_{451}\rangle, H_{915}=\langle\sigma_{27}, \sigma_{455}\rangle, H_{926}=\langle\sigma_{30}, \sigma_{128}\rangle, H_{930}=\langle\sigma_{30}, \sigma_{144}\rangle, H_{943}=\langle\sigma_{30}, \sigma_{451}\rangle, H_{944}=\langle\sigma_{30}, \sigma_{454}\rangle,$ $H_{1029}=\langle\sigma_{56}, \sigma_{137}\rangle, H_{1031}=\langle\sigma_{56}, \sigma_{144}\rangle, H_{1050}=\langle\sigma_{56}, \sigma_{539}\rangle, H_{1051}=\langle\sigma_{56}, \sigma_{546}\rangle, H_{1080}=\langle\sigma_{61}, \sigma_{128}\rangle, H_{1084}=\langle\sigma_{61}, \sigma_{144}\rangle,$ $H_{1093}=\langle\sigma_{61}, \sigma_{317}\rangle, H_{1094}=\langle\sigma_{61}, \sigma_{329}\rangle, H_{1106}=\langle\sigma_{68}, \sigma_{128}\rangle, H_{1108}=\langle\sigma_{68}, \sigma_{137}\rangle, H_{1119}=\langle\sigma_{68}, \sigma_{317}\rangle, H_{1120}=\langle\sigma_{68}, \sigma_{324}\rangle,$ $H_{1159}=\langle\sigma_{83}, \sigma_{128}\rangle, H_{1164}=\langle\sigma_{83}, \sigma_{144}\rangle, H_{1178}=\langle\sigma_{83}, \sigma_{416}\rangle, H_{1179}=\langle\sigma_{83}, \sigma_{424}\rangle, H_{1189}=\langle\sigma_{87}, \sigma_{137}\rangle, H_{1191}=\langle\sigma_{87}, \sigma_{144}\rangle,$ $H_{1197}=\langle\sigma_{87}, \sigma_{290}\rangle, H_{1198}=\langle\sigma_{87}, \sigma_{302}\rangle, H_{1207}=\langle\sigma_{95}, \sigma_{128}\rangle, H_{1209}=\langle\sigma_{95}, \sigma_{137}\rangle, H_{1220}=\langle\sigma_{95}, \sigma_{290}\rangle, H_{1221}=\langle\sigma_{95}, \sigma_{298}\rangle,$ $H_{1253}=\langle\sigma_{108}, \sigma_{128}\rangle, H_{1256}=\langle\sigma_{108}, \sigma_{137}\rangle, H_{1269}=\langle\sigma_{108}, \sigma_{416}\rangle, H_{1270}=\langle\sigma_{108}, \sigma_{429}\rangle, H_{1277}=\langle\sigma_{112}, \sigma_{137}\rangle,$ $H_{1279}=\langle\sigma_{112}, \sigma_{144}\rangle, H_{1285}=\langle\sigma_{112}, \sigma_{290}\rangle, H_{1286}=\langle\sigma_{112}, \sigma_{307}\rangle, H_{1295}=\langle\sigma_{120}, \sigma_{128}\rangle, H_{1299}=\langle\sigma_{120}, \sigma_{144}\rangle,$ $H_{1306}=\langle\sigma_{120}, \sigma_{290}\rangle, H_{1307}=\langle\sigma_{120}, \sigma_{299}\rangle$
16	$H_{1372}=\langle\sigma_2, \sigma_7, \sigma_{289}\rangle, H_{1373}=\langle\sigma_2, \sigma_7, \sigma_{593}\rangle, H_{1374}=\langle\sigma_2, \sigma_{17}, \sigma_{121}\rangle, H_{1375}=\langle\sigma_2, \sigma_{25}, \sigma_{127}\rangle, H_{1377}=\langle\sigma_2, \sigma_{25}, \sigma_{515}\rangle,$ $H_{1378}=\langle\sigma_2, \sigma_{55}, \sigma_{127}\rangle, H_{1380}=\langle\sigma_2, \sigma_{55}, \sigma_{527}\rangle, H_{1381}=\langle\sigma_2, \sigma_{83}, \sigma_{265}\rangle, H_{1382}=\langle\sigma_2, \sigma_{95}, \sigma_{391}\rangle, H_{1384}=\langle\sigma_3, \sigma_8, \sigma_{121}\rangle,$ $H_{1386}=\langle\sigma_3, \sigma_{22}, \sigma_{340}\rangle, H_{1387}=\langle\sigma_3, \sigma_{22}, \sigma_{445}\rangle, H_{1388}=\langle\sigma_3, \sigma_{25}, \sigma_{142}\rangle, H_{1389}=\langle\sigma_3, \sigma_{25}, \sigma_{392}\rangle, H_{1391}=\langle\sigma_3, \sigma_{56}, \sigma_{265}\rangle,$ $H_{1392}=\langle\sigma_3, \sigma_{61}, \sigma_{634}\rangle, H_{1393}=\langle\sigma_3, \sigma_{106}, \sigma_{142}\rangle, H_{1395}=\langle\sigma_3, \sigma_{106}, \sigma_{397}\rangle, H_{1398}=\langle\sigma_6, \sigma_8, \sigma_{121}\rangle, H_{1400}=\langle\sigma_6, \sigma_{15}, \sigma_{315}\rangle,$ $H_{1401}=\langle\sigma_6, \sigma_{15}, \sigma_{470}\rangle, H_{1402}=\langle\sigma_6, \sigma_{25}, \sigma_{135}\rangle, H_{1403}=\langle\sigma_6, \sigma_{25}, \sigma_{392}\rangle, H_{1405}=\langle\sigma_6, \sigma_{56}, \sigma_{265}\rangle, H_{1406}=\langle\sigma_6, \sigma_{68}, \sigma_{513}\rangle,$ $H_{1407}=\langle\sigma_6, \sigma_{81}, \sigma_{135}\rangle, H_{1409}=\langle\sigma_6, \sigma_{81}, \sigma_{404}\rangle, H_{1411}=\langle\sigma_7, \sigma_{27}, \sigma_{634}\rangle, H_{1412}=\langle\sigma_7, \sigma_{30}, \sigma_{513}\rangle, H_{1413}=\langle\sigma_7, \sigma_{81}, \sigma_{122}\rangle,$ $H_{1414}=\langle\sigma_7, \sigma_{81}, \sigma_{270}\rangle, H_{1416}=\langle\sigma_7, \sigma_{106}, \sigma_{122}\rangle, H_{1417}=\langle\sigma_7, \sigma_{106}, \sigma_{267}\rangle, H_{1425}=\langle\sigma_{15}, \sigma_{26}, \sigma_{391}\rangle,$ $H_{1426}=\langle\sigma_{15}, \sigma_{27}, \sigma_{634}\rangle, H_{1427}=\langle\sigma_{15}, \sigma_{55}, \sigma_{126}\rangle, H_{1428}=\langle\sigma_{15}, \sigma_{55}, \sigma_{266}\rangle, H_{1430}=\langle\sigma_{15}, \sigma_{106}, \sigma_{126}\rangle,$ $H_{1431}=\langle\sigma_{15}, \sigma_{106}, \sigma_{267}\rangle, H_{1437}=\langle\sigma_{22}, \sigma_{26}, \sigma_{391}\rangle, H_{1438}=\langle\sigma_{22}, \sigma_{30}, \sigma_{513}\rangle, H_{1439}=\langle\sigma_{22}, \sigma_{55}, \sigma_{123}\rangle,$ $H_{1440}=\langle\sigma_{22}, \sigma_{55}, \sigma_{266}\rangle, H_{1442}=\langle\sigma_{22}, \sigma_{81}, \sigma_{123}\rangle, H_{1443}=\langle\sigma_{22}, \sigma_{81}, \sigma_{270}\rangle$
18	$H_{385}=\langle\sigma_2, \sigma_{147}\rangle, H_{392}=\langle\sigma_2, \sigma_{181}\rangle, H_{410}=\langle\sigma_2, \sigma_{325}\rangle, H_{426}=\langle\sigma_2, \sigma_{543}\rangle, H_{455}=\langle\sigma_3, \sigma_{232}\rangle, H_{461}=\langle\sigma_3, \sigma_{310}\rangle,$ $H_{472}=\langle\sigma_3, \sigma_{428}\rangle, H_{527}=\langle\sigma_6, \sigma_{207}\rangle, H_{533}=\langle\sigma_6, \sigma_{303}\rangle, H_{544}=\langle\sigma_6, \sigma_{421}\rangle, H_{576}=\langle\sigma_7, \sigma_{316}\rangle, H_{582}=\langle\sigma_7, \sigma_{538}\rangle,$ $H_{703}=\langle\sigma_{15}, \sigma_{294}\rangle, H_{711}=\langle\sigma_{15}, \sigma_{420}\rangle, H_{792}=\langle\sigma_{22}, \sigma_{291}\rangle, H_{800}=\langle\sigma_{22}, \sigma_{417}\rangle, H_{844}=\langle\sigma_{25}, \sigma_{124}\rangle, H_{1008}=\langle\sigma_{55}, \sigma_{136}\rangle,$ $H_{1134}=\langle\sigma_{81}, \sigma_{132}\rangle, H_{1235}=\langle\sigma_{106}, \sigma_{129}\rangle$
	$H_{500}=\langle\sigma_4, \sigma_{451}\rangle, H_{502}=\langle\sigma_4, \sigma_{476}\rangle, H_{643}=\langle\sigma_9, \sigma_{290}\rangle, H_{645}=\langle\sigma_9, \sigma_{317}\rangle, H_{675}=\langle\sigma_{12}, \sigma_{290}\rangle, H_{679}=\langle\sigma_{12}, \sigma_{342}\rangle,$ $H_{726}=\langle\sigma_{16}, \sigma_{317}\rangle, H_{728}=\langle\sigma_{16}, \sigma_{342}\rangle, H_{952}=\langle\sigma_{31}, \sigma_{137}\rangle, H_{953}=\langle\sigma_{31}, \sigma_{144}\rangle, H_{972}=\langle\sigma_{39}, \sigma_{128}\rangle, H_{975}=\langle\sigma_{39}, \sigma_{144}\rangle,$ $H_{994}=\langle\sigma_{46}, \sigma_{128}\rangle, H_{995}=\langle\sigma_{46}, \sigma_{137}\rangle, H_{1053}=\langle\sigma_{57}, \sigma_{128}\rangle, H_{1054}=\langle\sigma_{57}, \sigma_{137}\rangle, H_{1070}=\langle\sigma_{60}, \sigma_{128}\rangle, H_{1072}=\langle\sigma_{60}, \sigma_{144}\rangle,$ $H_{1150}=\langle\sigma_{82}, \sigma_{137}\rangle, H_{1151}=\langle\sigma_{82}, \sigma_{144}\rangle$

位数	部分群
18	$H_{1397}=\langle\sigma_4, \sigma_{26}, \sigma_{122}\rangle, H_{1421}=\langle\sigma_8, \sigma_{26}, \sigma_{514}\rangle, H_{1422}=\langle\sigma_8, \sigma_{82}, \sigma_{266}\rangle, H_{1423}=\langle\sigma_9, \sigma_{108}, \sigma_{123}\rangle,$ $H_{1424}=\langle\sigma_{12}, \sigma_{83}, \sigma_{126}\rangle, H_{1434}=\langle\sigma_{16}, \sigma_{56}, \sigma_{122}\rangle, H_{1435}=\langle\sigma_{17}, \sigma_{30}, \sigma_{396}\rangle, H_{1436}=\langle\sigma_{17}, \sigma_{60}, \sigma_{270}\rangle,$ $H_{1446}=\langle\sigma_{24}, \sigma_{27}, \sigma_{393}\rangle, H_{1447}=\langle\sigma_{24}, \sigma_{57}, \sigma_{267}\rangle$
20	$H_{594}=\langle\sigma_8, \sigma_{33}\rangle, H_{595}=\langle\sigma_8, \sigma_{36}\rangle, H_{615}=\langle\sigma_8, \sigma_{273}\rangle, H_{616}=\langle\sigma_8, \sigma_{276}\rangle, H_{651}=\langle\sigma_{10}, \sigma_{26}\rangle, H_{652}=\langle\sigma_{10}, \sigma_{27}\rangle,$ $H_{657}=\langle\sigma_{10}, \sigma_{266}\rangle, H_{658}=\langle\sigma_{10}, \sigma_{267}\rangle, H_{660}=\langle\sigma_{11}, \sigma_{26}\rangle, H_{661}=\langle\sigma_{11}, \sigma_{30}\rangle, H_{666}=\langle\sigma_{11}, \sigma_{266}\rangle, H_{667}=\langle\sigma_{11}, \sigma_{270}\rangle,$ $H_{874}=\langle\sigma_{26}, \sigma_{159}\rangle, H_{875}=\langle\sigma_{26}, \sigma_{166}\rangle, H_{902}=\langle\sigma_{27}, \sigma_{151}\rangle, H_{903}=\langle\sigma_{27}, \sigma_{159}\rangle, H_{931}=\langle\sigma_{30}, \sigma_{151}\rangle, H_{934}=\langle\sigma_{30}, \sigma_{166}\rangle,$ $H_{957}=\langle\sigma_{33}, \sigma_{123}\rangle, H_{958}=\langle\sigma_{33}, \sigma_{127}\rangle, H_{962}=\langle\sigma_{36}, \sigma_{126}\rangle, H_{963}=\langle\sigma_{36}, \sigma_{127}\rangle, H_{967}=\langle\sigma_{37}, \sigma_{123}\rangle, H_{969}=\langle\sigma_{37}, \sigma_{135}\rangle,$ $H_{979}=\langle\sigma_{40}, \sigma_{122}\rangle, H_{980}=\langle\sigma_{40}, \sigma_{135}\rangle, H_{984}=\langle\sigma_{44}, \sigma_{126}\rangle, H_{986}=\langle\sigma_{44}, \sigma_{142}\rangle, H_{989}=\langle\sigma_{45}, \sigma_{122}\rangle, H_{991}=\langle\sigma_{45}, \sigma_{142}\rangle,$ $H_{1036}=\langle\sigma_{56}, \sigma_{177}\rangle, H_{1037}=\langle\sigma_{56}, \sigma_{180}\rangle, H_{1059}=\langle\sigma_{58}, \sigma_{122}\rangle, H_{1060}=\langle\sigma_{58}, \sigma_{123}\rangle, H_{1064}=\langle\sigma_{59}, \sigma_{122}\rangle, H_{1065}=\langle\sigma_{59}, \sigma_{126}\rangle$
24	$H_{365}=\langle\sigma_2, \sigma_9\rangle, H_{371}=\langle\sigma_2, \sigma_{39}\rangle, H_{373}=\langle\sigma_2, \sigma_{57}\rangle, H_{394}=\langle\sigma_2, \sigma_{201}\rangle, H_{401}=\langle\sigma_2, \sigma_{279}\rangle, H_{412}=\langle\sigma_2, \sigma_{393}\rangle,$ $H_{435}=\langle\sigma_3, \sigma_{31}\rangle, H_{450}=\langle\sigma_3, \sigma_{175}\rangle, H_{457}=\langle\sigma_3, \sigma_{271}\rangle, H_{507}=\langle\sigma_6, \sigma_{31}\rangle, H_{522}=\langle\sigma_6, \sigma_{175}\rangle, H_{529}=\langle\sigma_6, \sigma_{271}\rangle,$ $H_{563}=\langle\sigma_7, \sigma_{145}\rangle, H_{691}=\langle\sigma_{15}, \sigma_{145}\rangle, H_{780}=\langle\sigma_{22}, \sigma_{145}\rangle$ $H_{396}=\langle\sigma_2, \sigma_{209}\rangle, H_{397}=\langle\sigma_2, \sigma_{213}\rangle, H_{406}=\langle\sigma_2, \sigma_{305}\rangle, H_{451}=\langle\sigma_3, \sigma_{182}\rangle, H_{452}=\langle\sigma_3, \sigma_{187}\rangle, H_{464}=\langle\sigma_3, \sigma_{349}\rangle,$ $H_{524}=\langle\sigma_6, \sigma_{182}\rangle, H_{525}=\langle\sigma_6, \sigma_{187}\rangle, H_{537}=\langle\sigma_6, \sigma_{326}\rangle, H_{565}=\langle\sigma_7, \sigma_{148}\rangle, H_{566}=\langle\sigma_7, \sigma_{149}\rangle, H_{692}=\langle\sigma_{15}, \sigma_{148}\rangle,$ $H_{693}=\langle\sigma_{15}, \sigma_{149}\rangle, H_{782}=\langle\sigma_{22}, \sigma_{148}\rangle, H_{783}=\langle\sigma_{22}, \sigma_{149}\rangle$ $H_{488}=\langle\sigma_4, \sigma_{127}\rangle, H_{495}=\langle\sigma_4, \sigma_{289}\rangle, H_{498}=\langle\sigma_4, \sigma_{415}\rangle, H_{602}=\langle\sigma_8, \sigma_{145}\rangle, H_{611}=\langle\sigma_8, \sigma_{201}\rangle, H_{646}=\langle\sigma_9, \sigma_{340}\rangle,$ $H_{649}=\langle\sigma_9, \sigma_{658}\rangle, H_{677}=\langle\sigma_{12}, \sigma_{315}\rangle, H_{681}=\langle\sigma_{12}, \sigma_{537}\rangle, H_{724}=\langle\sigma_{16}, \sigma_{289}\rangle, H_{730}=\langle\sigma_{16}, \sigma_{415}\rangle, H_{740}=\langle\sigma_{17}, \sigma_{145}\rangle,$ $H_{747}=\langle\sigma_{17}, \sigma_{175}\rangle, H_{814}=\langle\sigma_{24}, \sigma_{145}\rangle, H_{821}=\langle\sigma_{24}, \sigma_{175}\rangle$ $H_{489}=\langle\sigma_4, \sigma_{128}\rangle, H_{496}=\langle\sigma_4, \sigma_{290}\rangle, H_{499}=\langle\sigma_4, \sigma_{416}\rangle, H_{647}=\langle\sigma_9, \sigma_{342}\rangle, H_{650}=\langle\sigma_9, \sigma_{660}\rangle, H_{678}=\langle\sigma_{12}, \sigma_{317}\rangle,$ $H_{682}=\langle\sigma_{12}, \sigma_{539}\rangle, H_{725}=\langle\sigma_{16}, \sigma_{290}\rangle, H_{731}=\langle\sigma_{16}, \sigma_{416}\rangle, H_{951}=\langle\sigma_{31}, \sigma_{128}\rangle, H_{974}=\langle\sigma_{39}, \sigma_{137}\rangle, H_{997}=\langle\sigma_{46}, \sigma_{144}\rangle,$ $H_{1055}=\langle\sigma_{57}, \sigma_{144}\rangle, H_{1071}=\langle\sigma_{60}, \sigma_{137}\rangle, H_{1149}=\langle\sigma_{82}, \sigma_{128}\rangle$ $H_{624}=\langle\sigma_8, \sigma_{323}\rangle, H_{628}=\langle\sigma_8, \sigma_{348}\rangle, H_{755}=\langle\sigma_{17}, \sigma_{296}\rangle, H_{876}=\langle\sigma_{26}, \sigma_{186}\rangle, H_{877}=\langle\sigma_{26}, \sigma_{191}\rangle, H_{907}=\langle\sigma_{27}, \sigma_{187}\rangle,$ $H_{908}=\langle\sigma_{27}, \sigma_{213}\rangle, H_{935}=\langle\sigma_{30}, \sigma_{182}\rangle, H_{937}=\langle\sigma_{30}, \sigma_{212}\rangle, H_{1033}=\langle\sigma_{56}, \sigma_{161}\rangle, H_{1035}=\langle\sigma_{56}, \sigma_{168}\rangle, H_{1087}=\langle\sigma_{61}, \sigma_{209}\rangle,$ $H_{1113}=\langle\sigma_{68}, \sigma_{197}\rangle, H_{1166}=\langle\sigma_{83}, \sigma_{152}\rangle, H_{1214}=\langle\sigma_{95}, \sigma_{170}\rangle$ $H_{659}=\langle\sigma_{10}, \sigma_{317}\rangle, H_{668}=\langle\sigma_{11}, \sigma_{342}\rangle, H_{771}=\langle\sigma_{18}, \sigma_{290}\rangle, H_{959}=\langle\sigma_{33}, \sigma_{137}\rangle, H_{964}=\langle\sigma_{36}, \sigma_{144}\rangle, H_{968}=\langle\sigma_{37}, \sigma_{128}\rangle,$ $H_{981}=\langle\sigma_{40}, \sigma_{144}\rangle, H_{985}=\langle\sigma_{44}, \sigma_{128}\rangle, H_{990}=\langle\sigma_{45}, \sigma_{137}\rangle, H_{1061}=\langle\sigma_{58}, \sigma_{137}\rangle, H_{1066}=\langle\sigma_{59}, \sigma_{144}\rangle, H_{1099}=\langle\sigma_{63}, \sigma_{144}\rangle,$ $H_{1125}=\langle\sigma_{70}, \sigma_{137}\rangle, H_{1180}=\langle\sigma_{84}, \sigma_{128}\rangle, H_{1226}=\langle\sigma_{96}, \sigma_{128}\rangle$
36	$H_{501}=\langle\sigma_4, \sigma_{452}\rangle, H_{607}=\langle\sigma_8, \sigma_{161}\rangle, H_{609}=\langle\sigma_8, \sigma_{197}\rangle, H_{644}=\langle\sigma_9, \sigma_{293}\rangle, H_{676}=\langle\sigma_{12}, \sigma_{292}\rangle, H_{727}=\langle\sigma_{16}, \sigma_{318}\rangle,$ $H_{743}=\langle\sigma_{17}, \sigma_{152}\rangle, H_{745}=\langle\sigma_{17}, \sigma_{170}\rangle, H_{817}=\langle\sigma_{24}, \sigma_{152}\rangle, H_{819}=\langle\sigma_{24}, \sigma_{170}\rangle$ $H_{608}=\langle\sigma_8, \sigma_{162}\rangle, H_{610}=\langle\sigma_8, \sigma_{198}\rangle, H_{744}=\langle\sigma_{17}, \sigma_{154}\rangle, H_{746}=\langle\sigma_{17}, \sigma_{172}\rangle, H_{818}=\langle\sigma_{24}, \sigma_{155}\rangle, H_{820}=\langle\sigma_{24}, \sigma_{173}\rangle,$ $H_{884}=\langle\sigma_{26}, \sigma_{454}\rangle, H_{1044}=\langle\sigma_{56}, \sigma_{324}\rangle, H_{1172}=\langle\sigma_{83}, \sigma_{298}\rangle, H_{1263}=\langle\sigma_{108}, \sigma_{299}\rangle$ $H_{621}=\langle\sigma_8, \sigma_{316}\rangle, H_{631}=\langle\sigma_8, \sigma_{538}\rangle, H_{753}=\langle\sigma_{17}, \sigma_{294}\rangle, H_{764}=\langle\sigma_{17}, \sigma_{420}\rangle, H_{827}=\langle\sigma_{24}, \sigma_{291}\rangle, H_{836}=\langle\sigma_{24}, \sigma_{417}\rangle,$ $H_{864}=\langle\sigma_{26}, \sigma_{124}\rangle, H_{1028}=\langle\sigma_{56}, \sigma_{136}\rangle, H_{1160}=\langle\sigma_{83}, \sigma_{132}\rangle, H_{1254}=\langle\sigma_{108}, \sigma_{129}\rangle$
48	$H_{382}=\langle\sigma_2, \sigma_{123}\rangle, H_{395}=\langle\sigma_2, \sigma_{207}\rangle, H_{403}=\langle\sigma_2, \sigma_{291}\rangle, H_{405}=\langle\sigma_2, \sigma_{303}\rangle, H_{415}=\langle\sigma_2, \sigma_{417}\rangle, H_{416}=\langle\sigma_2, \sigma_{421}\rangle,$ $H_{453}=\langle\sigma_3, \sigma_{188}\rangle, H_{465}=\langle\sigma_3, \sigma_{350}\rangle, H_{480}=\langle\sigma_3, \sigma_{664}\rangle, H_{523}=\langle\sigma_6, \sigma_{181}\rangle, H_{536}=\langle\sigma_6, \sigma_{325}\rangle, H_{552}=\langle\sigma_6, \sigma_{543}\rangle,$ $H_{564}=\langle\sigma_7, \sigma_{146}\rangle, H_{694}=\langle\sigma_{15}, \sigma_{150}\rangle, H_{781}=\langle\sigma_{22}, \sigma_{147}\rangle$ $H_{623}=\langle\sigma_8, \sigma_{321}\rangle, H_{627}=\langle\sigma_8, \sigma_{346}\rangle, H_{754}=\langle\sigma_{17}, \sigma_{295}\rangle, H_{872}=\langle\sigma_{26}, \sigma_{154}\rangle, H_{873}=\langle\sigma_{26}, \sigma_{155}\rangle, H_{904}=\langle\sigma_{27}, \sigma_{163}\rangle,$ $H_{905}=\langle\sigma_{27}, \sigma_{167}\rangle, H_{932}=\langle\sigma_{30}, \sigma_{158}\rangle, H_{933}=\langle\sigma_{30}, \sigma_{162}\rangle, H_{1032}=\langle\sigma_{56}, \sigma_{159}\rangle, H_{1034}=\langle\sigma_{56}, \sigma_{166}\rangle, H_{1086}=\langle\sigma_{61}, \sigma_{185}\rangle,$ $H_{1112}=\langle\sigma_{68}, \sigma_{192}\rangle, H_{1165}=\langle\sigma_{83}, \sigma_{151}\rangle, H_{1213}=\langle\sigma_{95}, \sigma_{169}\rangle$
60	$H_{485}=\langle\sigma_4, \sigma_{31}\rangle, H_{492}=\langle\sigma_4, \sigma_{175}\rangle, H_{494}=\langle\sigma_4, \sigma_{271}\rangle, H_{640}=\langle\sigma_9, \sigma_{145}\rangle, H_{672}=\langle\sigma_{12}, \sigma_{145}\rangle, H_{721}=\langle\sigma_{16}, \sigma_{145}\rangle$ $H_{603}=\langle\sigma_8, \sigma_{145}\rangle, H_{604}=\langle\sigma_8, \sigma_{149}\rangle, H_{741}=\langle\sigma_{17}, \sigma_{148}\rangle, H_{742}=\langle\sigma_{17}, \sigma_{149}\rangle, H_{815}=\langle\sigma_{24}, \sigma_{148}\rangle, H_{816}=\langle\sigma_{24}, \sigma_{149}\rangle$
72	$H_{389}=\langle\sigma_2, \sigma_{161}\rangle, H_{393}=\langle\sigma_2, \sigma_{197}\rangle, H_{409}=\langle\sigma_2, \sigma_{323}\rangle, H_{419}=\langle\sigma_2, \sigma_{453}\rangle, H_{447}=\langle\sigma_3, \sigma_{152}\rangle, H_{449}=\langle\sigma_3, \sigma_{170}\rangle,$ $H_{460}=\langle\sigma_3, \sigma_{296}\rangle, H_{519}=\langle\sigma_6, \sigma_{152}\rangle, H_{521}=\langle\sigma_6, \sigma_{170}\rangle, H_{532}=\langle\sigma_6, \sigma_{296}\rangle$
120	$H_{370}=\langle\sigma_2, \sigma_{33}\rangle, H_{388}=\langle\sigma_2, \sigma_{159}\rangle, H_{391}=\langle\sigma_2, \sigma_{177}\rangle, H_{400}=\langle\sigma_2, \sigma_{273}\rangle, H_{446}=\langle\sigma_3, \sigma_{151}\rangle, H_{518}=\langle\sigma_6, \sigma_{151}\rangle$ $H_{605}=\langle\sigma_8, \sigma_{154}\rangle, H_{606}=\langle\sigma_8, \sigma_{155}\rangle, H_{655}=\langle\sigma_{10}, \sigma_{148}\rangle, H_{656}=\langle\sigma_{10}, \sigma_{149}\rangle, H_{664}=\langle\sigma_{11}, \sigma_{148}\rangle, H_{665}=\langle\sigma_{11}, \sigma_{149}\rangle$
360	$H_{491}=\langle\sigma_4, \sigma_{152}\rangle$
720	$H_{387}=\langle\sigma_2, \sigma_{153}\rangle$

6次対称群の部分群のうち、可移である以下の15個の共役類について、交換子群の列を示す。

H_{94} (位数6) の場合: $\{H_{94}, H_1$ (単位群) $\}$

H_{1321} (位数6) の場合: $\{H_{1321}, H_{301}$ (位数3, H_{88} の共役類), H_1 (単位群) $\}$

H_{619} (位数12) の場合: $\{H_{619}, H_{598}$ (位数4), H_1 (単位群) $\}$

H_{622} (位数12) の場合: $\{H_{622}, H_{325}$ (位数3, H_{88} の共役類), H_1 (単位群) $\}$

H_{500} (位数18) の場合: $\{H_{500}, H_{89}$ (位数3, H_{88} の共役類), H_1 (単位群) $\}$

H_{396} (位数24) の場合: $\{H_{396}, H_{1040}$ (位数4, H_{598} の共役類), H_1 (単位群) $\}$

H_{624} (位数24) の場合: $\{H_{624}, H_{758}$ (位数12, H_{619} の共役類), H_{738} (位数4, H_{598} の共役類), H_1 (単位群) $\}$

H_{659} (位数24) の場合: $\{H_{659}, H_{758}$ (位数12, H_{619} の共役類), H_{738} (位数4, H_{598} の共役類), H_1 (単位群) $\}$

H_{501} (位数36) の場合: $\{H_{501}, H_{490}$ (位数9), H_1 (単位群) $\}$

H_{608} (位数36) の場合: $\{H_{608}, H_{1153}$ (位数9, H_{490} の共役類), H_1 (単位群) $\}$

H_{623} (位数48) の場合: $\{H_{623}, H_{758}$ (位数12, H_{619} の共役類), H_{738} (位数4, H_{598} の共役類), H_1 (単位群) $\}$

H_{603} (位数60) の場合: $\{H_{603}\}$

H_{389} (位数72) の場合: $\{H_{389}, H_{1422}$ (位数18, H_{1397} の共役類), H_{1153} (位数9, H_{490} の共役類), H_1 (単位群) $\}$

H_{491} (位数360) の場合: $\{H_{491}\}$

H_{387} (位数720) の場合: $\{H_{387}, H_{491}$ (位数360) $\}$

これより、可移で可解な部分群(既約で可解な6次方程式のガロア群に対応する)は、 H_{94} (位数6),

H_{1321} (位数6), H_{619} (位数12), H_{622} (位数12), H_{500} (位数18), H_{396} (位数24), H_{624} (位数24),

H_{659} (位数24), H_{501} (位数36), H_{608} (位数36), H_{623} (位数48), H_{389} (位数72) とそれらの共役類である。

5. ガロア群の組成列

有限群 G が代数方程式のガロア群である場合に、その組成列 $G=G_0 \supset G_1 \supset \dots \supset G_s$ を求める方法を示す。ここで、 $G_k (k=1, 2, \dots, s)$ は G_{k-1} の真部分群で最大の正規部分群とする。最後の G_s は単位群である。 k 番目の組成列 $G_{k-1} \supset G_k$ において、 G_{k-1} から G_k を求めるには、第1節で示した部分群の計算法において、部分群を求めるところを正規部分群を求めるように変更するだけでよい。これには、手順(1)において、 σ_i から生成される部分群 $\langle \sigma_i \rangle$ を、 σ_i を含む最小の正規部分群 $\langle\langle \sigma_i \rangle\rangle$ (本稿だけで用いる表記である)に変更するだけでよい。これには、まず $H=\{\sigma_i\}$ とし、以下の2つの処理を繰り返すことにより、 H に元を追加していく。

(1) $\sigma^{-1}\tau\sigma (\sigma \in G_{k-1}, \tau \in H)$ で H に含まれないものがあれば、これを H に追加する。

(2) $\tau\tau' (\tau, \tau' \in H)$ で H に含まれないものがあれば、これを H に追加する。

H に追加される元がなくなれば、上の2つの処理を終了する。このときの H を $\langle\langle \sigma_i \rangle\rangle$ とする。手順(2), (3)は変更する必要はない。正規部分群から生成される部分群は必ず正規部分群となるからである。

以下に示すのは、与えられた有限群の組成列 $G[0], G[1], G[2], \dots$ を求めるアルゴリズムである。

与えられた有限群を $G[0]$ とする。

```
for (k=1; |G[k-1]| > 1; k++) {
    G[k-1]のすべての正規部分群H[1], H[2], ..., H[c]を求める。
    G[k] = {σi};
    for (i=1; i ≤ c; i++) {
        if (|G[k-1]| > |H[i]| && |H[i]| > |G[k]|) G[k] = H[i];
    }
}
```

(例) $G=H_{1397}$ (6次対称群の部分群の1つ)の場合

この場合は $n=6, N=18$ で、 G は以下のように表される。

$$G = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6, \sigma_7, \sigma_8, \sigma_9, \sigma_{10}, \sigma_{11}, \sigma_{12}, \sigma_{13}, \sigma_{14}, \sigma_{15}, \sigma_{16}, \sigma_{17}, \sigma_{18}\}$$

$$= \{\{1, 2, 3, 4, 5, 6\}, \{1, 2, 3, 5, 6, 4\}, \{1, 2, 3, 6, 4, 5\}, \{1, 3, 2, 4, 6, 5\}, \{1, 3, 2, 5, 4, 6\},$$

$$\{1, 3, 2, 6, 5, 4\}, \{2, 1, 3, 4, 6, 5\}, \{2, 1, 3, 5, 4, 6\}, \{2, 1, 3, 6, 5, 4\}, \{2, 3, 1, 4, 5, 6\},$$

$$\{2, 3, 1, 5, 6, 4\}, \{2, 3, 1, 6, 4, 5\}, \{3, 1, 2, 4, 5, 6\}, \{3, 1, 2, 5, 6, 4\}, \{3, 1, 2, 6, 4, 5\},$$

$$\{3, 2, 1, 4, 6, 5\}, \{3, 2, 1, 5, 4, 6\}, \{3, 2, 1, 6, 5, 4\}\}$$

《第1段階》

G_0 の正規部分群は以下の7個である。

$$H_1 = \langle\langle \sigma_1 \rangle\rangle = \{\sigma_1\}, \quad H_2 = \langle\langle \sigma_2 \rangle\rangle = \{\sigma_1, \sigma_2, \sigma_3\}$$

$$H_3 = \langle\langle \sigma_4 \rangle\rangle = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6, \sigma_7, \sigma_8, \sigma_9, \sigma_{10}, \sigma_{11}, \sigma_{12}, \sigma_{13}, \sigma_{14}, \sigma_{15}, \sigma_{16}, \sigma_{17}, \sigma_{18}\}$$

$$H_4 = \langle\langle \sigma_{10} \rangle\rangle = \{\sigma_1, \sigma_{10}, \sigma_{13}\}, \quad H_5 = \langle\langle \sigma_{11} \rangle\rangle = \{\sigma_1, \sigma_{11}, \sigma_{15}\}, \quad H_6 = \langle\langle \sigma_{12} \rangle\rangle = \{\sigma_1, \sigma_{12}, \sigma_{14}\}$$

$$H_7 = \langle\langle \sigma_2, \sigma_{10} \rangle\rangle = \{\sigma_1, \sigma_2, \sigma_3, \sigma_{10}, \sigma_{11}, \sigma_{12}, \sigma_{13}, \sigma_{14}, \sigma_{15}\}$$

これより、 $G_1 = H_7 = \langle\langle \sigma_2, \sigma_{10} \rangle\rangle = \{\sigma_1, \sigma_2, \sigma_3, \sigma_{10}, \sigma_{11}, \sigma_{12}, \sigma_{13}, \sigma_{14}, \sigma_{15}\}$ とする。

《第2段階》

G_1 の正規部分群は以下の6個である。

$$H_1 = \langle\langle \sigma_1 \rangle\rangle = \{\sigma_1\}, \quad H_2 = \langle\langle \sigma_2 \rangle\rangle = \{\sigma_1, \sigma_2, \sigma_3\}, \quad H_3 = \langle\langle \sigma_{10} \rangle\rangle = \{\sigma_1, \sigma_{10}, \sigma_{13}\}, \quad H_4 = \langle\langle \sigma_{11} \rangle\rangle = \{\sigma_1, \sigma_{11}, \sigma_{15}\}$$

$$H_5 = \langle\langle \sigma_{12} \rangle\rangle = \{\sigma_1, \sigma_{12}, \sigma_{14}\}, \quad H_6 = \langle\langle \sigma_2, \sigma_{10} \rangle\rangle = \{\sigma_1, \sigma_2, \sigma_3, \sigma_{10}, \sigma_{11}, \sigma_{12}, \sigma_{13}, \sigma_{14}, \sigma_{15}\}$$

これより, $G_2 = H_2 = \langle\langle \sigma_2 \rangle\rangle = \{\sigma_1, \sigma_2, \sigma_3\}$ とする。

《第3段階》

G_2 の正規部分群は以下の2個である。

$$H_1 = \langle\langle \sigma_1 \rangle\rangle = \{\sigma_1\}, \quad H_2 = \langle\langle \sigma_2 \rangle\rangle = \{\sigma_1, \sigma_2, \sigma_3\}$$

これより, $G_3 = H_1 = \langle\langle \sigma_1 \rangle\rangle = \{\sigma_1\}$ とする。

以上より, 組成列は以下のようになる。

$$G_0 \supset G_1 \supset G_2 \supset G_3$$

ここで

$$G_0 = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6, \sigma_7, \sigma_8, \sigma_9, \sigma_{10}, \sigma_{11}, \sigma_{12}, \sigma_{13}, \sigma_{14}, \sigma_{15}, \sigma_{16}, \sigma_{17}, \sigma_{18}\}$$

$$G_1 = \{\sigma_1, \sigma_2, \sigma_3, \sigma_{10}, \sigma_{11}, \sigma_{12}, \sigma_{13}, \sigma_{14}, \sigma_{15}\}$$

$$G_2 = \{\sigma_1, \sigma_2, \sigma_3\}$$

$$G_3 = \{\sigma_1\}$$

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